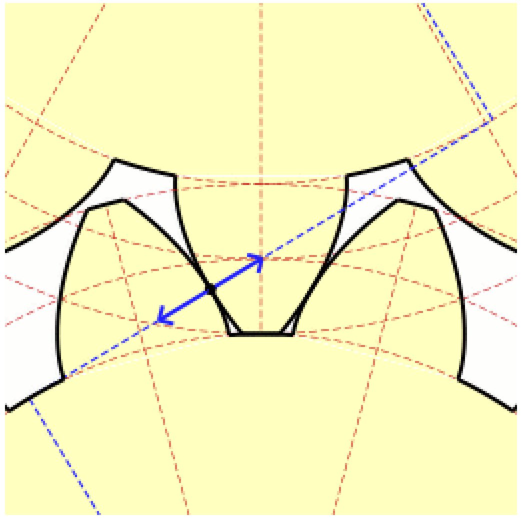
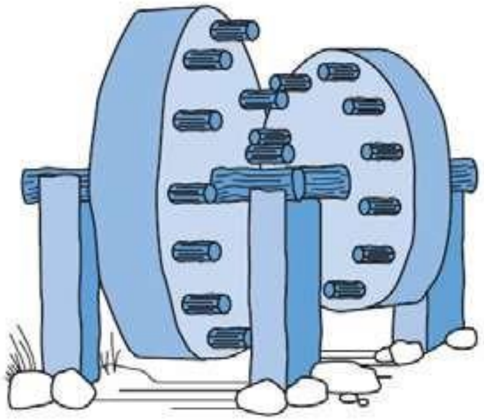


GEAR.....

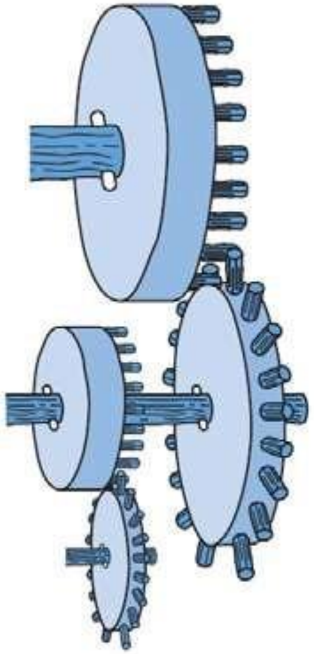
- **Power transmission is the movement of energy from its place of generation to a location where it is applied to performing useful work**

- **A gear is a component within a transmission device that transmits rotational force to another gear or device.**

PRIMITIVE GEARS



Right-angle gearing



Parallel gearing

GEAR MATERIALS

- The materials used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc.
- The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze.
- The non-metallic materials like wood, rawhide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.
- The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important.
- The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness.
- The phosphor bronze is widely used for worms gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

TYPES OF GEARS

1. According to the position of axes of the shafts.

a. Parallel

1.Spur Gear

2.Helical Gear

3.Rack and Pinion

b. Intersecting

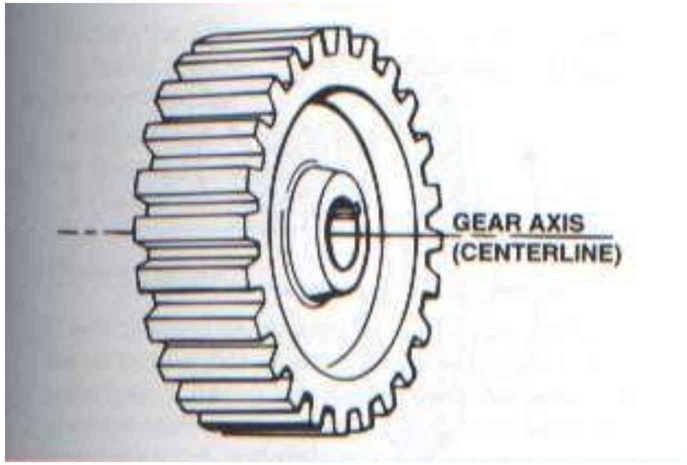
Bevel Gear

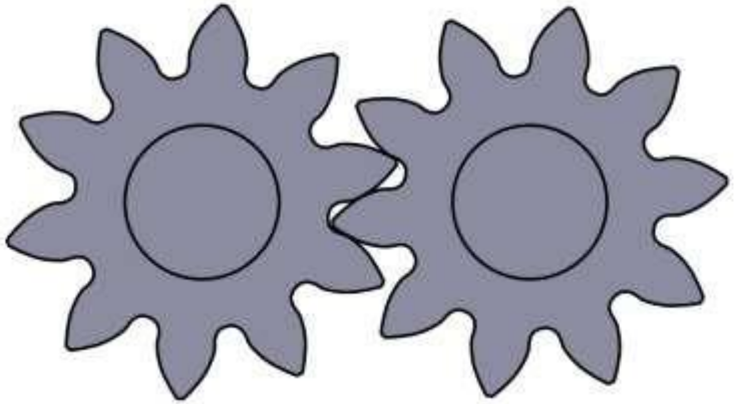
c. Non-intersecting and Non-parallel

worm and worm gears

SPUR GEAR

- Teeth is parallel to axis of rotation
- Transmit power from one shaft to another parallel shaft
- Used in Electric screwdriver, oscillating sprinkler, windup alarm clock, washing machine and clothes dryer





SPUR GEAR

Advantages

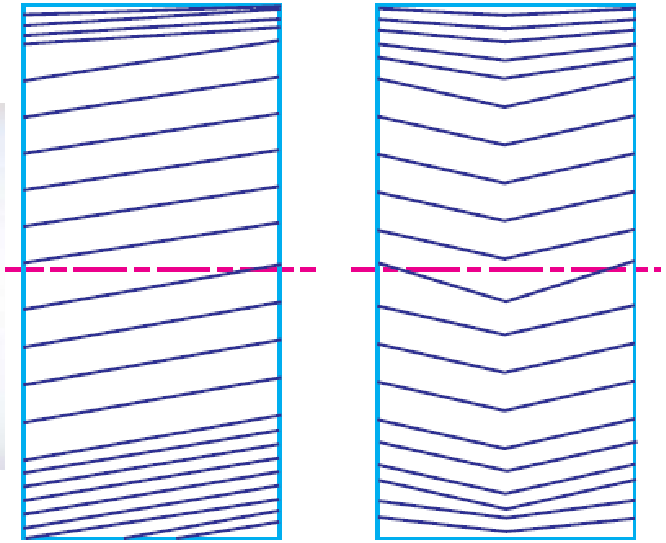
- They offer constant velocity ratio
- Spur gears are highly reliable
- Spur gears are simplest, hence easiest to design and manufacture
- A spur gear is more efficient if you compare it with helical gear of same size
- Spur gear teeth are parallel to its axis. Hence, spur gear train does not produce axial thrust. So the gear shafts can be mounted easily using ball bearings.
- They can be used to transmit large amount of power (of the order of 50,000 kW)

Disadvantages

- Spur gear are slow-speed gears
- Gear teeth experience a large amount of stress
- They cannot transfer power between non-parallel shafts
- They cannot be used for long distance power transmission.
- Spur gears produce a lot of noise when operating at high speeds.
- when compared with other types of gears, they are not as strong as them

HELICAL GEAR

- The teeth on helical gears are cut at an angle to the face of the gear
- This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears
- One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees



HELICAL GEAR

- Advantages

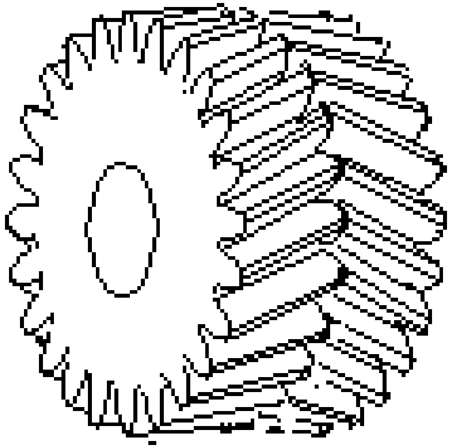
- The angled teeth engage more gradually than do spur gear teeth causing them to run more smoothly and quietly
- Helical gears are highly durable and are ideal for high load applications.
- At any given time their load is distributed over several teeth, resulting in less wear
- Can transmit motion and power between either parallel or right angle shafts

- Disadvantages

- •An obvious disadvantage of the helical gears is a resultant thrust along the axis of the gear, which needs to be accommodated by appropriate thrust bearings, and a greater degree of sliding friction between the meshing teeth, often addressed with additives in the lubricant.
- Thus we can say that helical gears cause losses due to the unique geometry along the axis of the helical gear's shaft.
- •Efficiency of helical gear is less because helical gear trains have sliding contacts between the teeth which in turns produce axial thrust of gear shafts and generate more heat. So, more power loss and less efficiency

HERRINGBONE GEARS

- To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces

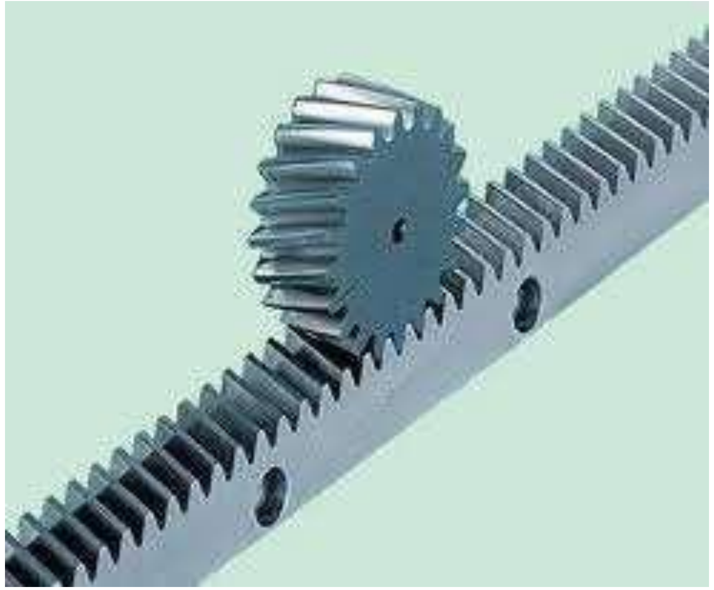


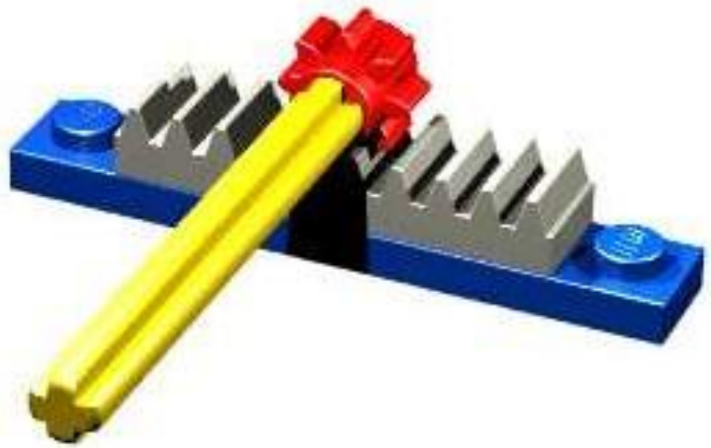
- Herringbone gears are mostly used on heavy machinery.

RACK AND PINION

- **Rack and pinion gears** are used to convert rotation (From the pinion) into linear motion (of the rack)

- A perfect example of this is the steering system on many cars





RACK AND PINION

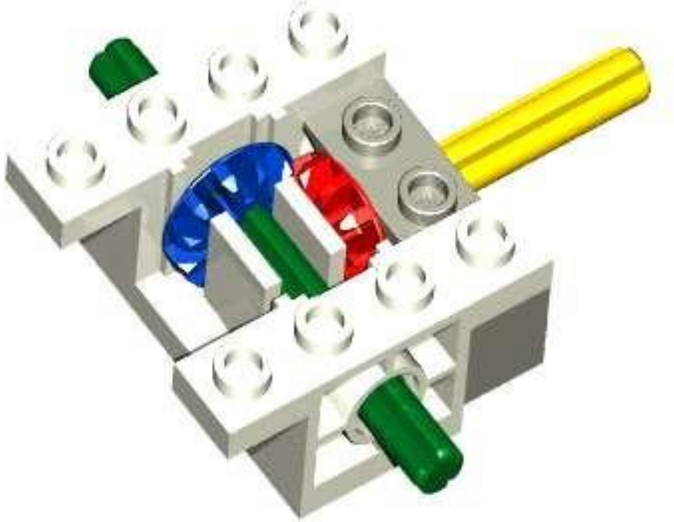
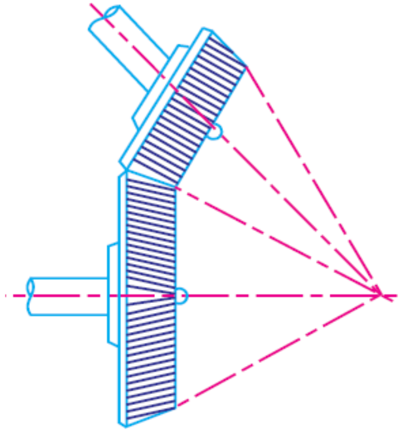
- **Advantages**

- Cheap
- Compact
- Robust
- Easiest way to convert rotation motion into linear motion
- Rack and pinion gives easier and more compact control over the vehicle

- **Disadvantages**
- Since being the most ancient, the wheel is also the most convenient and somewhat more extensive in terms of energy too. Due to the apparent friction, you would already have guessed just how much of the power being input gives in terms of output, a lot of the force applied to the mechanism is burned up in overcoming friction, to be more precise somewhat around 80% of the overall force is burned to overcome one.
- The rack and pinion can only work with certain levels of friction. Too high a friction and the mechanism will be subject to wear more than usual and will require more force to operate.
- The most adverse disadvantage of rack and pinion would also be due to the inherent friction, the same force that actually makes things work in the mechanism. Due to the friction, it is under a constant wear, possibly needing replacement after a certain time

BEVEL GEARS

- **Bevel gears** are useful when the direction of a shaft's rotation needs to be changed
- They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well
- The teeth on bevel gears can be **straight**, **spiral** or **hypoid**
- locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.

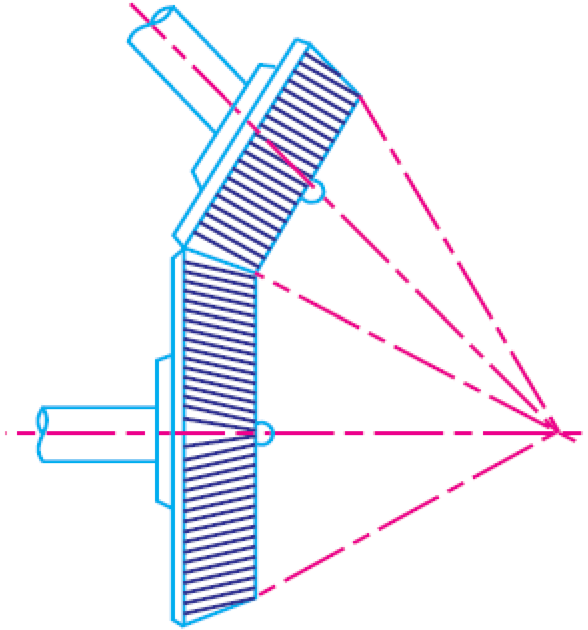




BEVEL GEARS

- **Advantages**
- Worm gear drives operate silently and smoothly.
- They are self-locking.
- They occupy less space.
- They have good meshing effectiveness.
- They can be used for reducing speed and increasing torque.
- High velocity ratio of the order of 100 can be obtained in a single step

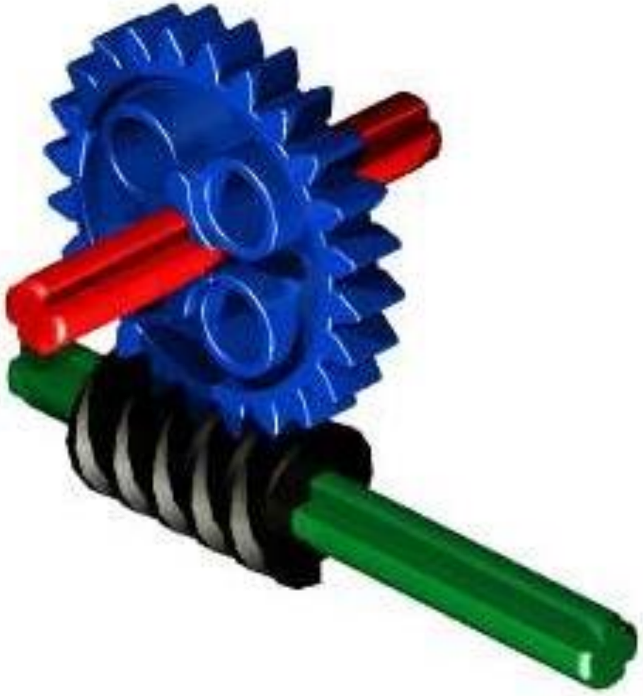
- **Disadvantages**
- Worm gear materials are expensive.
- Worm drives have high power losses
- A disadvantage is the potential for considerable sliding action, leading to low efficiency
- They produce a lot of heat.

STRAIGHT AND SPIRAL BEVEL GEARS

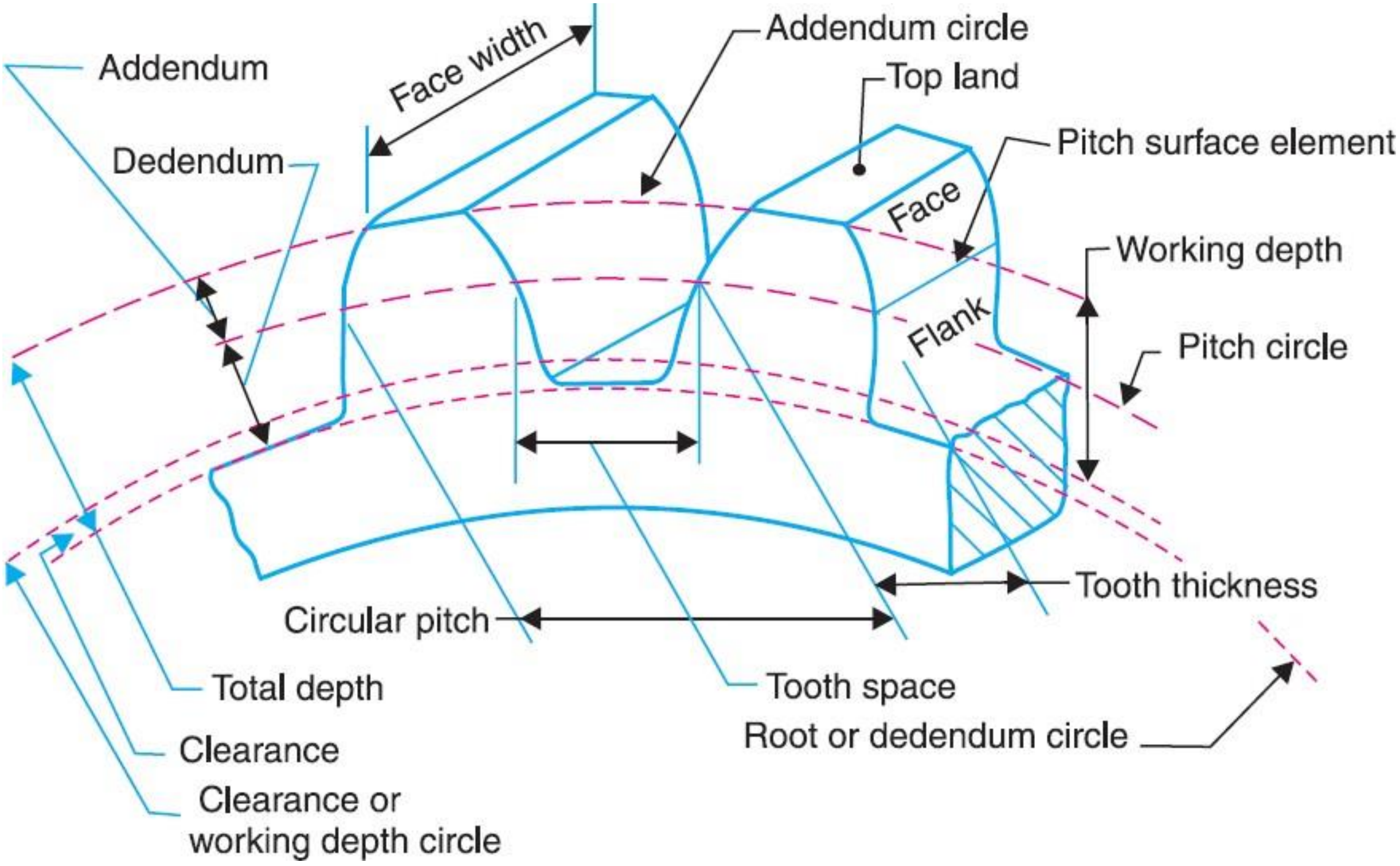


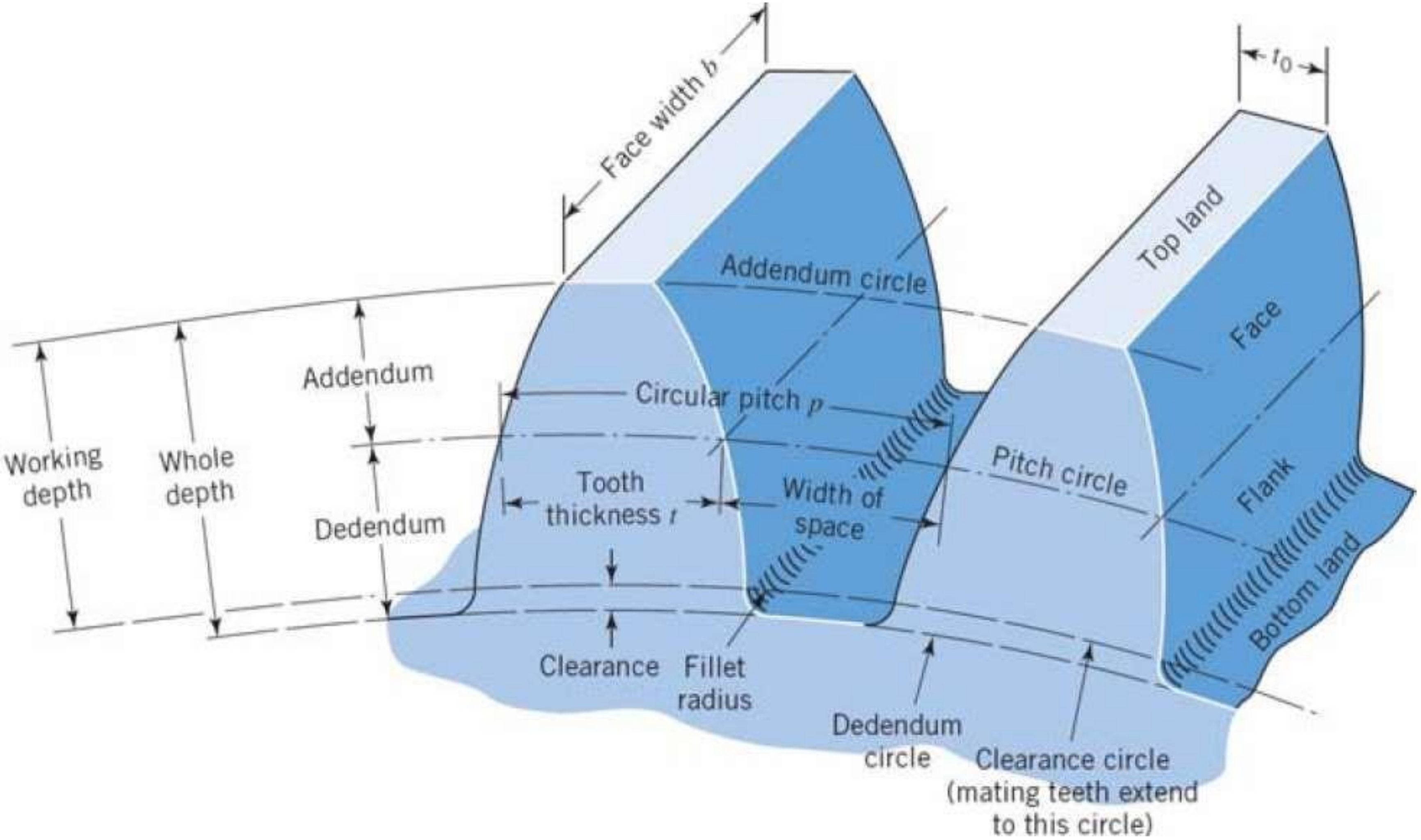
WORM AND WORM GEAR

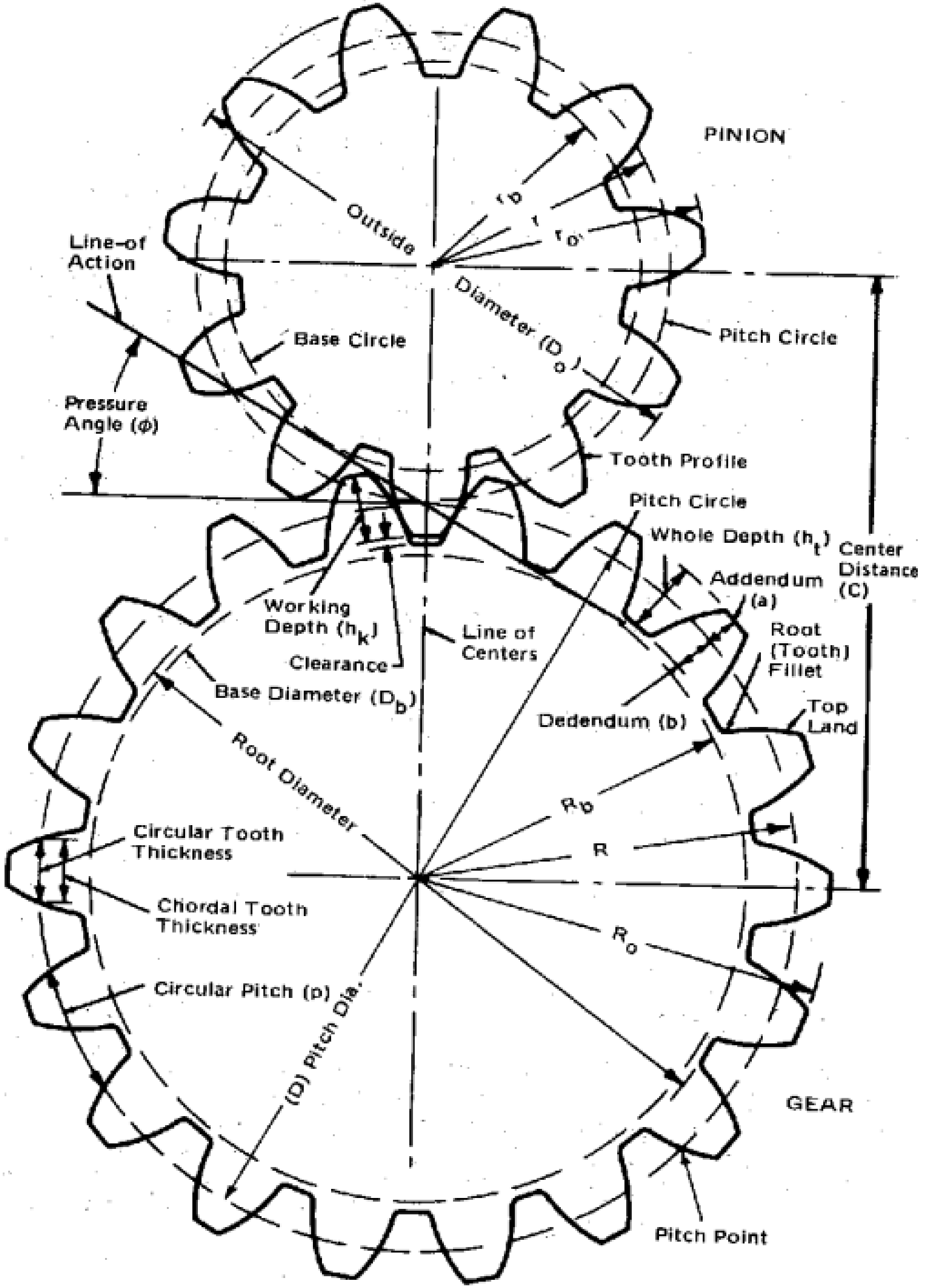
- **Worm gears** are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater
- Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm
- Worm gears are used widely in material handling and transportation machinery, machine tools, automobiles etc



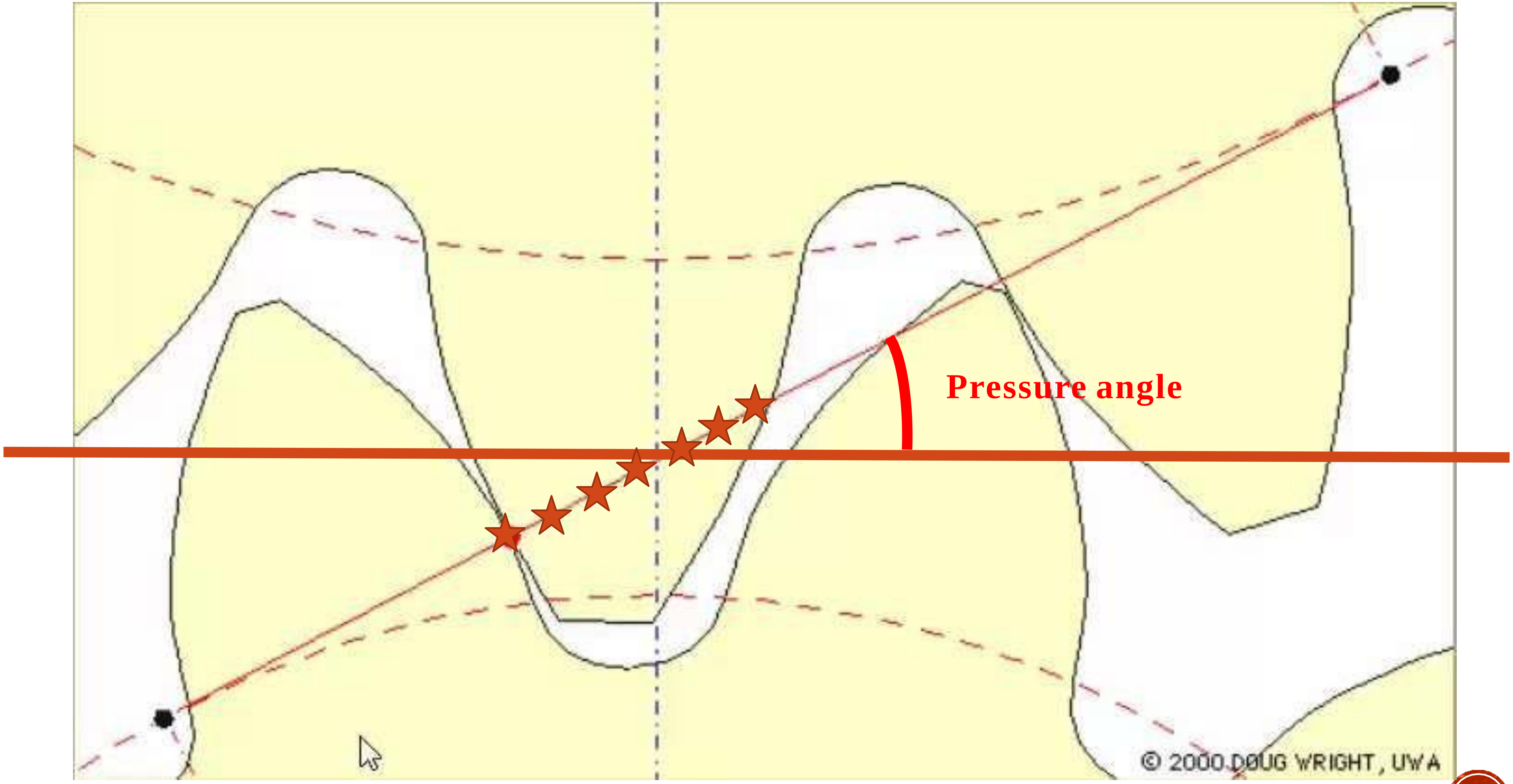
NOMENCLATURE OF SPUR GEARS




Face width b
t_0
Top land
Face
Addendum circle
Addendum
Circular pitch p
Pitch circle
Working depth
Whole depth
Tooth thickness t
Width of space
Flank
Dedendum
Bottom land
Clearance
Fillet radius
Dedendum circle
Clearance circle (mating teeth extend to this circle)


PINION
Outside
Diameter (D_o)
r_b
r
r_o
Line-of Action
Base Circle
Pitch Circle
Pressure Angle (ϕ)
Tooth Profile
Pitch Circle
Whole Depth (h_t)
Center Distance (C)
Addendum (a)
Working Depth (h_k)
Clearance
Line of Centers
Root (Tooth) Fillet
Base Diameter (D_b)
Top Land
Dedendum (b)
Root Diameter
R_b
R
Circular Tooth Thickness
Chordal Tooth Thickness
R_o
Circular Pitch (p)
(D) Pitch Dia.
GEAR
Pitch Point

- ***Pitch circle.*** *It is an imaginary circle which by pure rolling action would give the same* motion as the actual gear.
- ***Pitch circle diameter.*** *It is the diameter of the pitch circle. The size of the gear is usually* specified by the pitch circle diameter. It is also known as *pitch diameter.*
- ***Pitch point.*** *It is a common point of contact between two pitch circles.*
- ***Pitch surface.*** *It is the surface of the rolling discs which the meshing gears have replaced* at the pitch circle.
- ***Pressure angle or angle of obliquity.*** *It is the angle between the common normal to two* gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are 14 1/2 ° and 20°.


Pressure angle
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- ***Addendum.*** *It is the radial distance of a tooth from the pitch circle to the top of the tooth.*
- ***Dedendum.*** *It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.*
- ***Addendum circle.*** *It is the circle drawn through the top of the teeth and is concentric with* the pitch circle.
- ***Dedendum circle.*** *It is the circle drawn through the bottom of the teeth. It is also called* root circle.

Note : Root circle diameter =
Pitch circle diameter × cos φ ,
where φ is the pressure angle.

Circular pitch. *It is the distance measured on the circumference of the pitch circle from* a point of one tooth to the corresponding point on the next tooth. It is usually denoted by P_c ,Mathematically,

$$\text{Circular pitch,} \quad p_c = \pi D/T$$

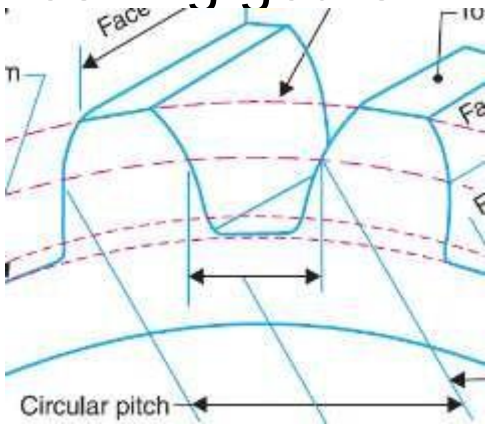
where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

- A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If *D*1 and $D2$ are the diameters of the two meshing gears having the teeth $T1$ and $T2$ respectively, then f mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$



Diametral pitch. *It is the ratio of number of teeth to the pitch circle diameter in millimetres.* It is denoted by p_d.
M

$$\text{Diametral pitch,} \quad p_d = \frac{T}{D} = \frac{\pi}{p_c} \quad \ldots \left(\because\ p_c = \frac{\pi D}{T} \right)$$

where T = Number of teeth, and

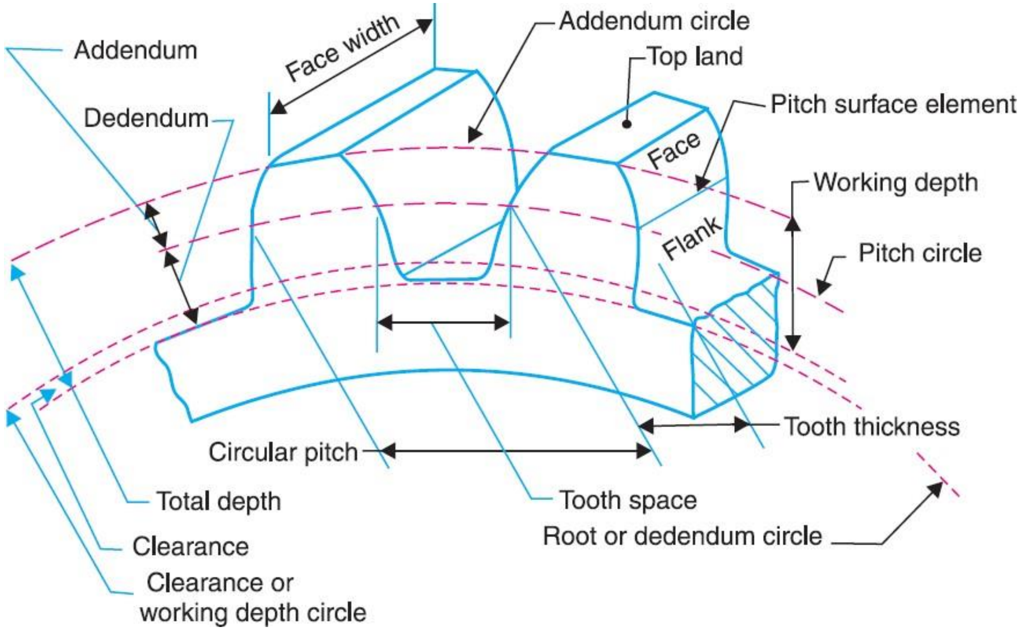
D = Pitch circle diameter.

Module. *It is the ratio of the pitch circle diameter in millimeters to the number of teeth.* It is usually denoted by m. *Mathematically,* Module, $m = D/T$

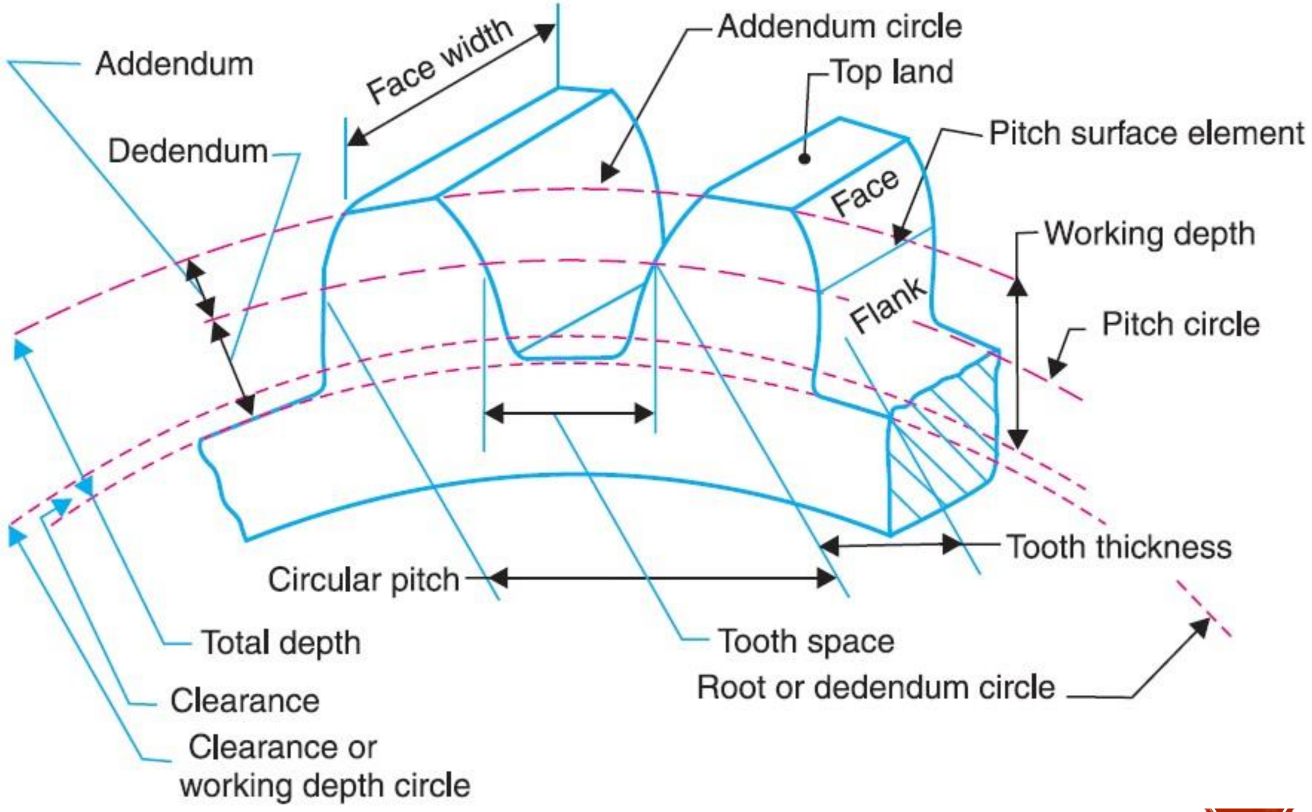
Clearance. *It is the radial distance from the top of the tooth to the bottom of the tooth, in* a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle.***

Total Depth. *It is the radial distance between the addendum and the dedendum circles of* a gear. It is equal to the sum of

- ***Working depth.*** *It is the radial distance from the addendum circle to the clearance circle.* It is equal to the sum of the addendum of the two meshing gears.
- ***Tooth thickness.*** *It is the width of the tooth measured along the pitch circle.*
- ***Tooth space .*** *It is the width of space between the two adjacent teeth measured along the pitch* circle.
- ***Backlash.*** *It is the difference between the tooth space and the tooth thickness, as measured* along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.



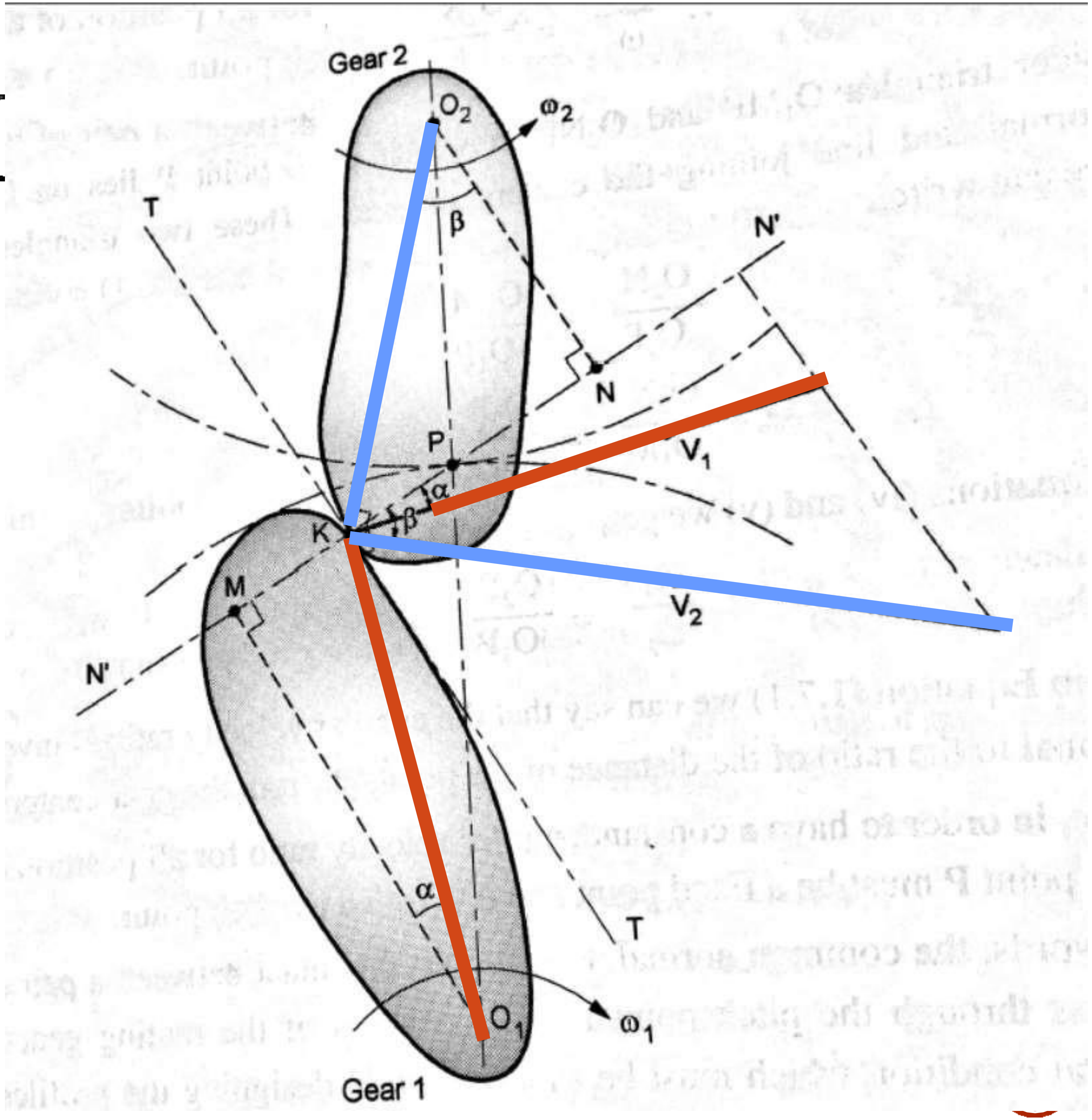
- ***Face of tooth.*** *It is the surface of the gear tooth above the pitch surface.*
- ***Flank of tooth.*** *It is the surface of the gear tooth below the pitch surface.*
- ***Top land.*** *It is the surface of the top of the tooth.*
- ***Face width.*** *It is the width of the gear tooth measured parallel to its axis.*
- ***Profile.*** *It is the curve formed by the face and flank of the tooth.*
- ***Fillet radius.*** *It is the radius that connects the root circle to the profile of the tooth.*

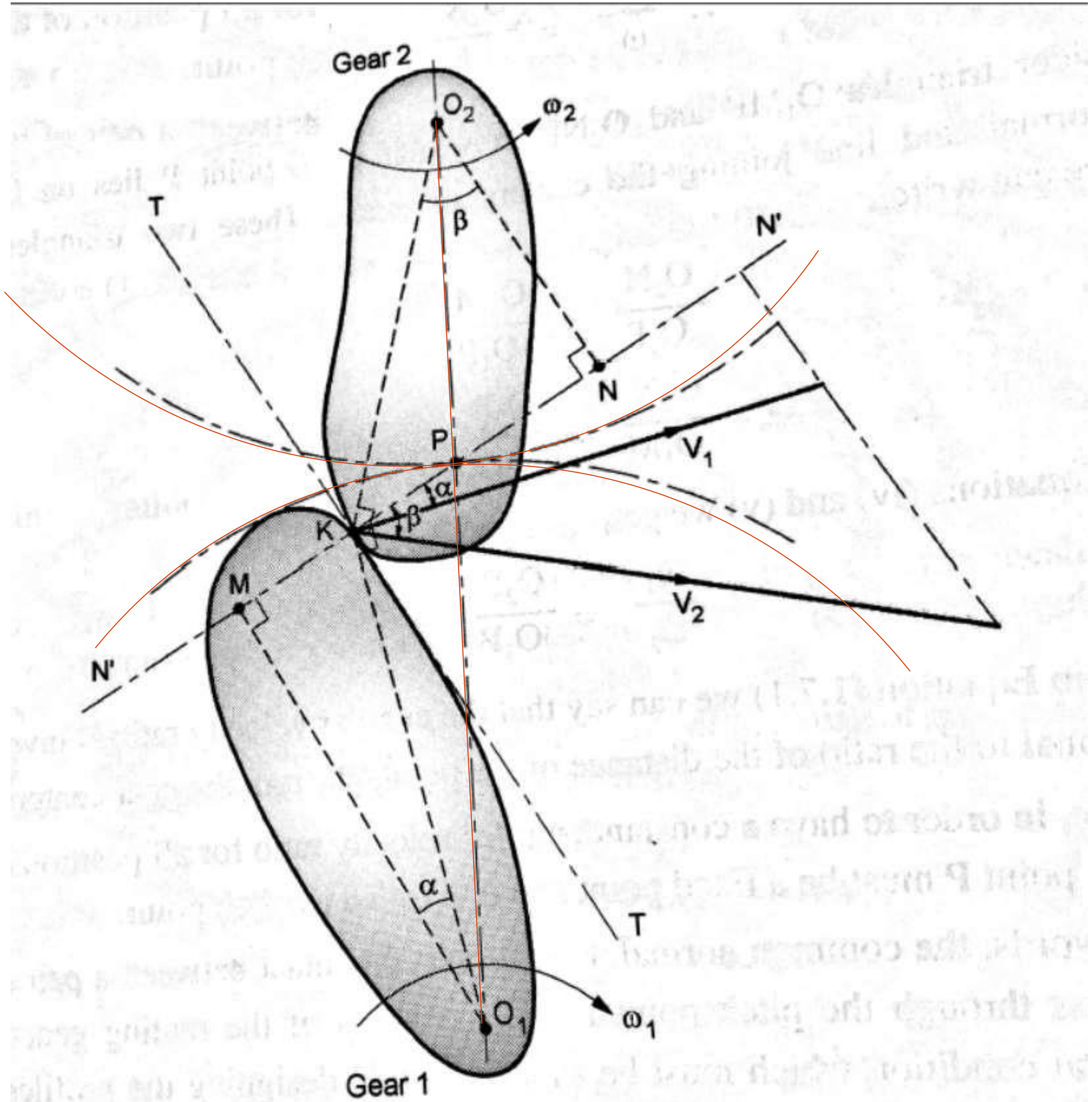


LAW OF GEARI

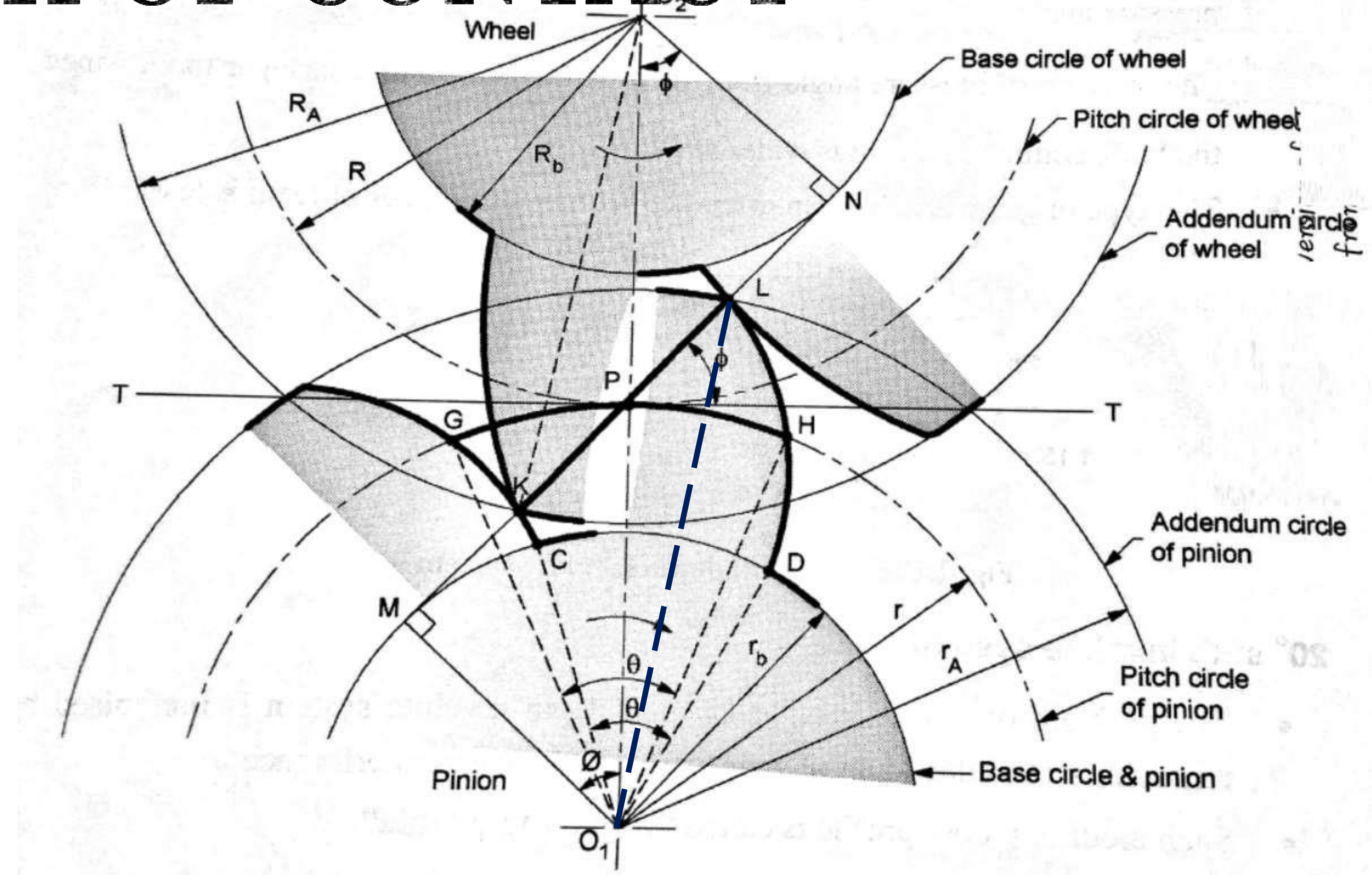
- ***The common normal at the point of contact between a pair of teeth must always pass through the pitch point.***

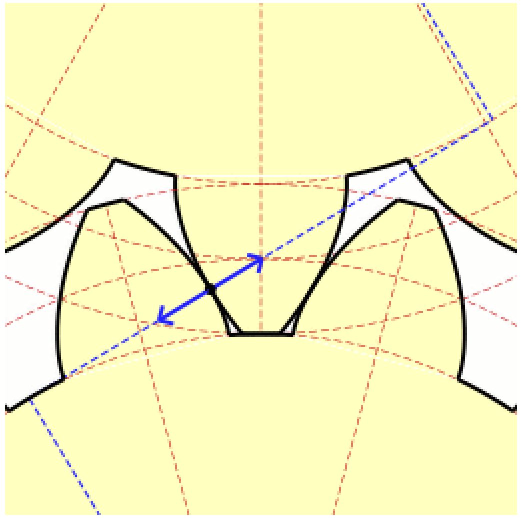
- The angular velocity ratio between two gears of a gear set must remain constant throughout the mesh.

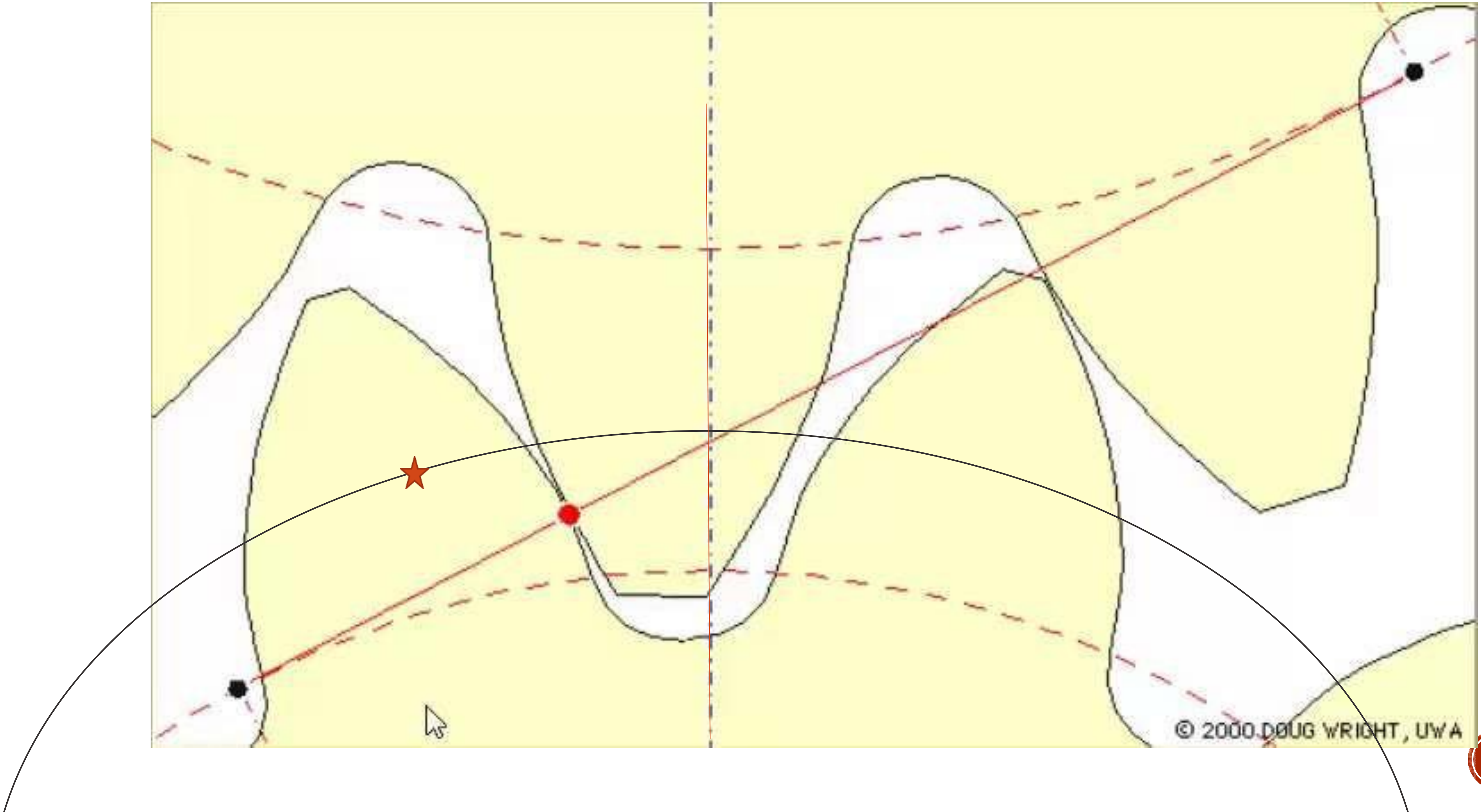



Gear 2
O_2
ω_2
T
β
N'
N
P
V_1
α
β
K
M
N'
V_2
α
T
O_1
ω_1
Gear 1

PATH OF CONTACT



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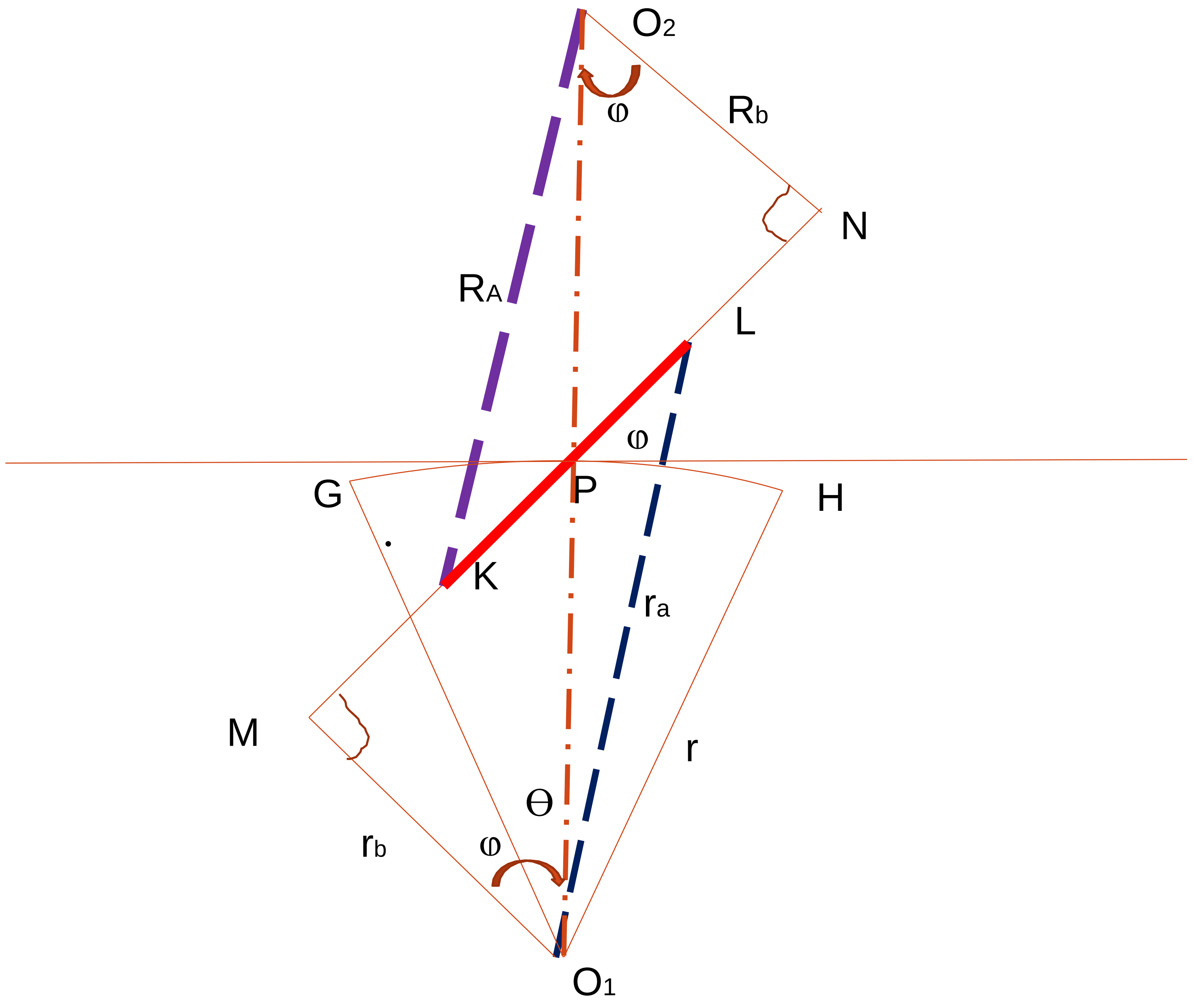
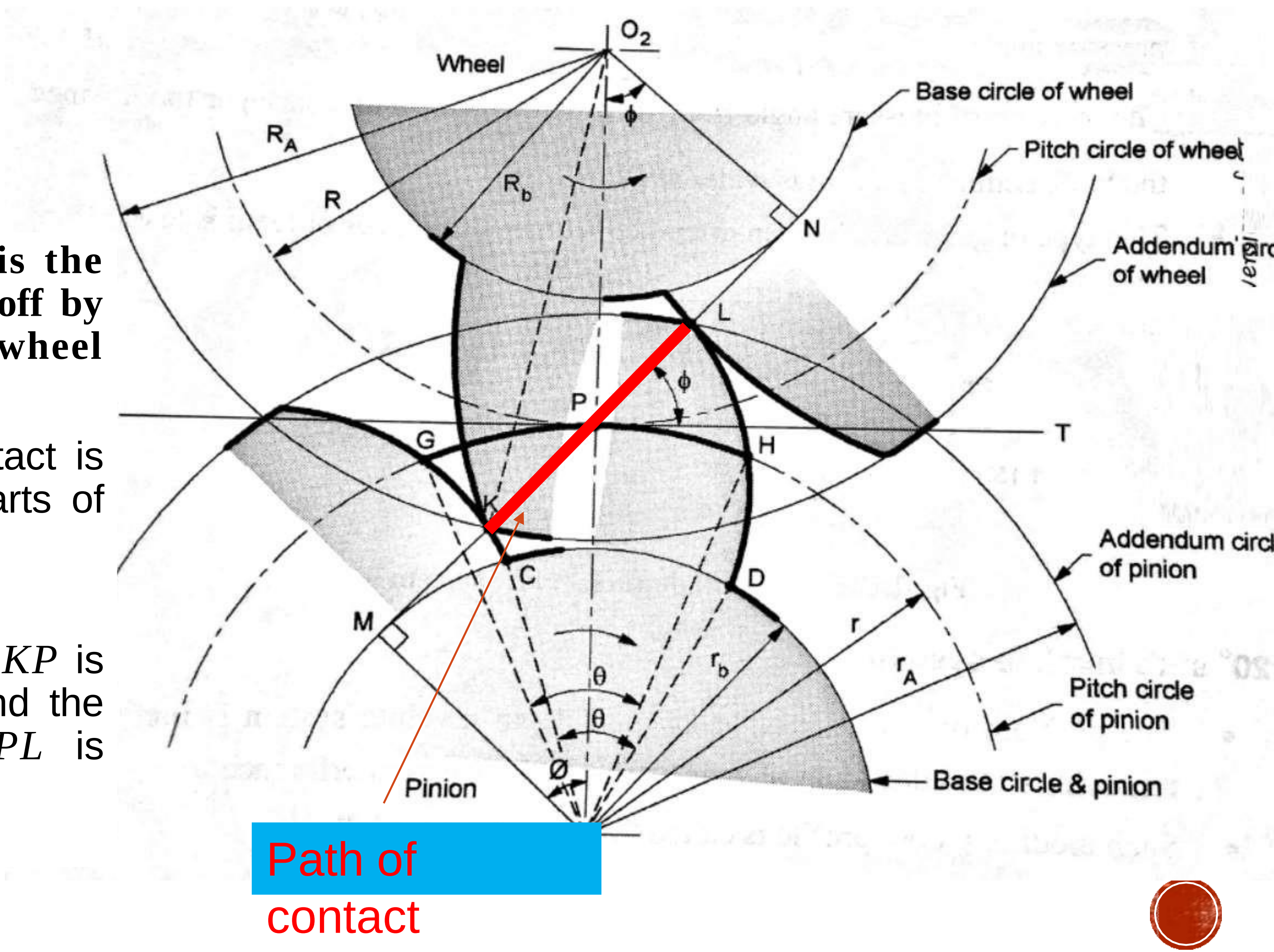
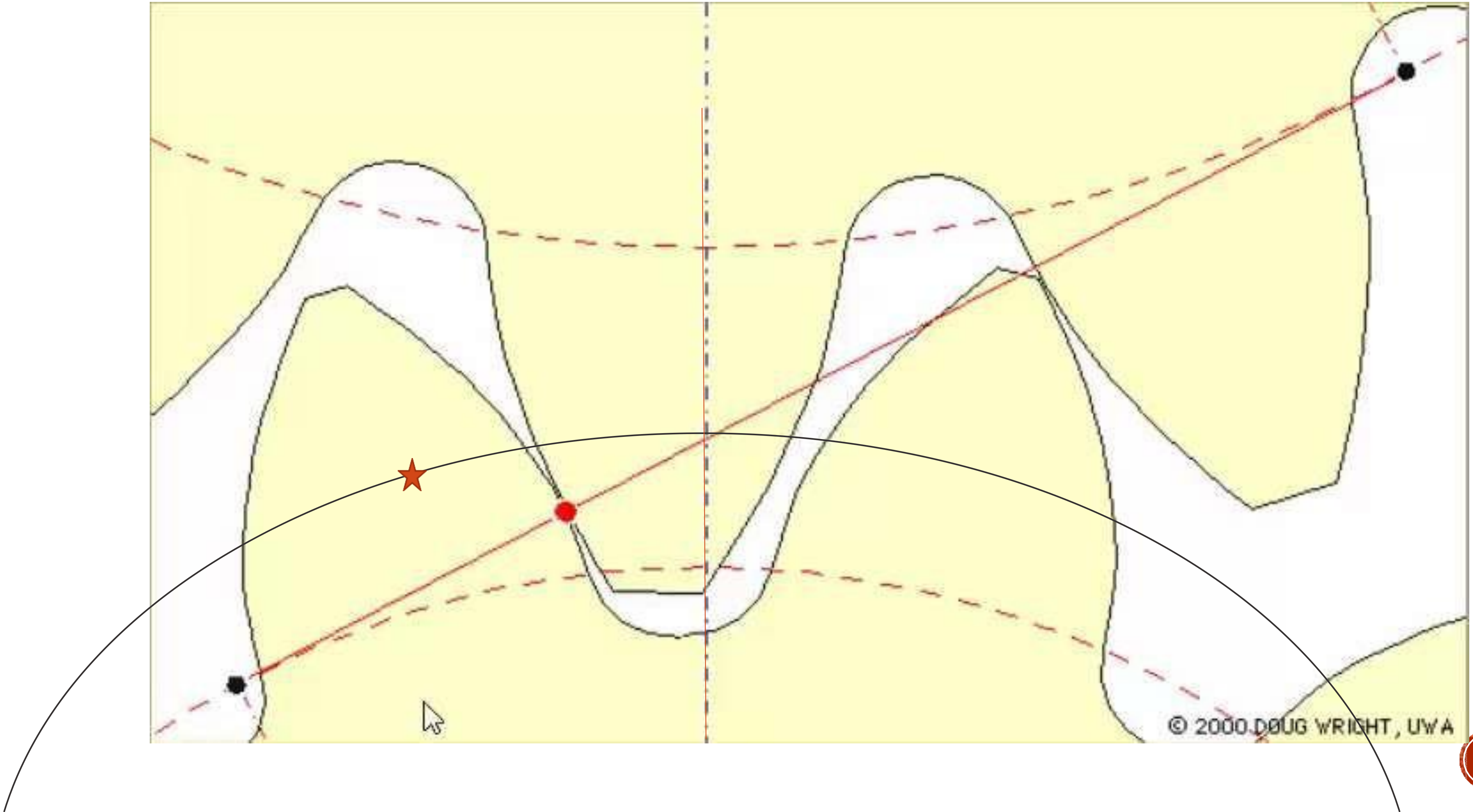
PATH OF CONTACT



PATH OF CONTACT

- **The length of path of contact is the length of common normal cut off by the addendum circles of the wheel and the pinion.**
- Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL.
- The part of the path of contact KP is known as ***path of approach*** and the part of the path of contact PL is known as ***path of recess***.

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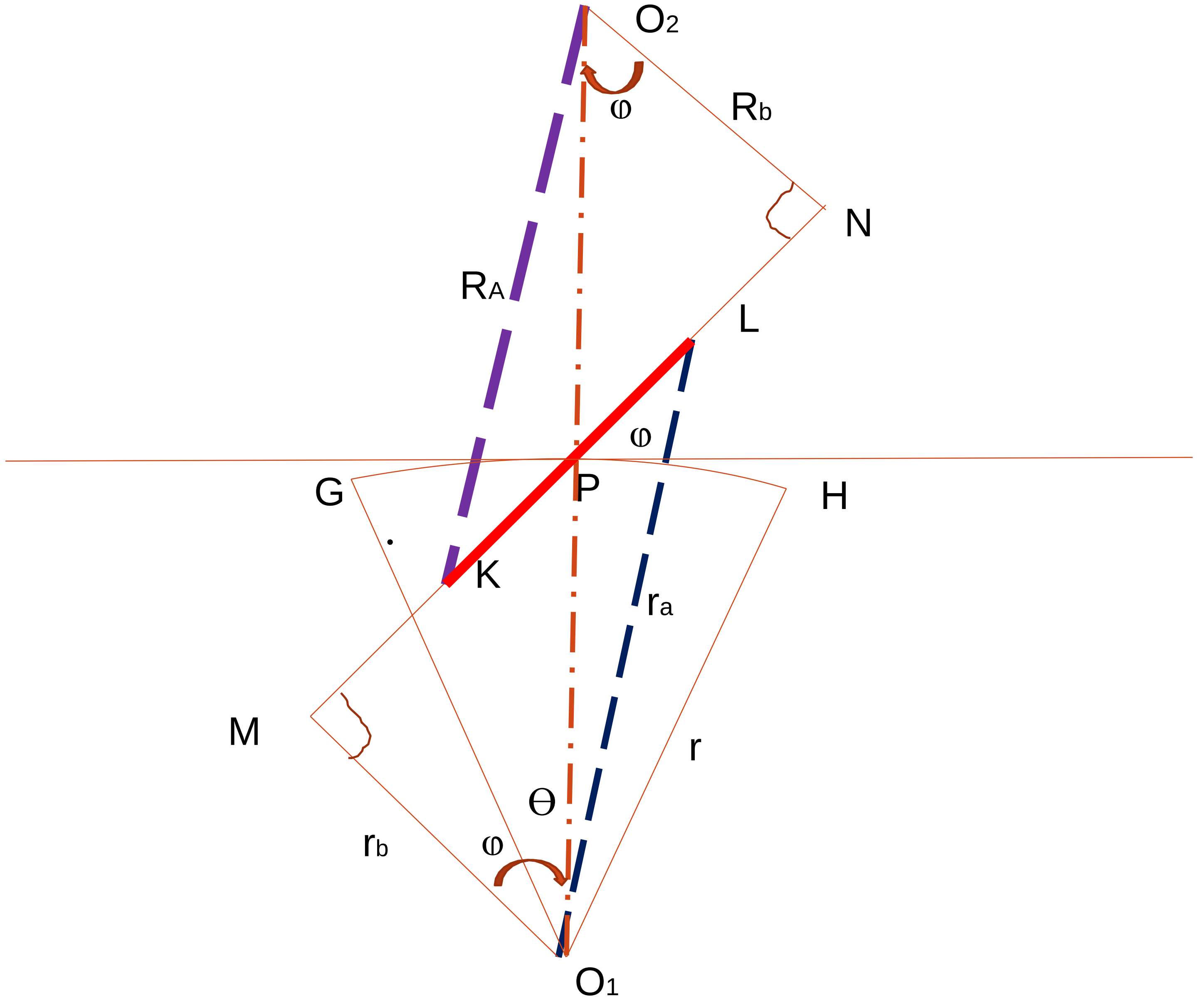
PATH OF CONTACT



PATH OFCONTACT

- **radius of the base circle of pinion,**

$$O_1M = O_1P \cos\phi = r\cos\phi$$

- **radius of the base circle of wheel,**

$$O_2N = O_2P\cos\phi = R\cos\phi$$

- **Now from right angled triangle *O*2*KN*,**

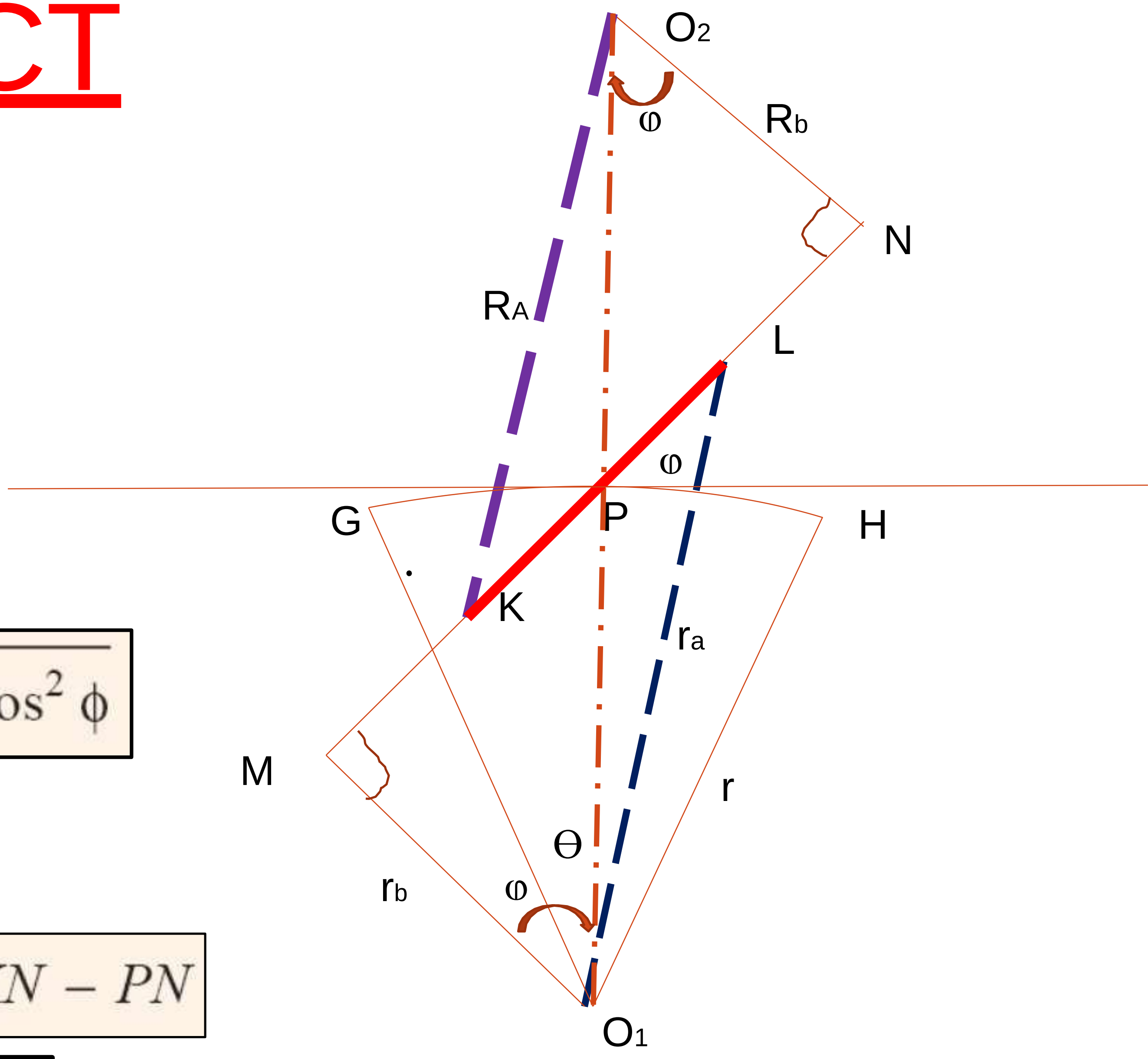
$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$\text{and} \quad PN = O_2P\sin\phi = R\sin\phi$$

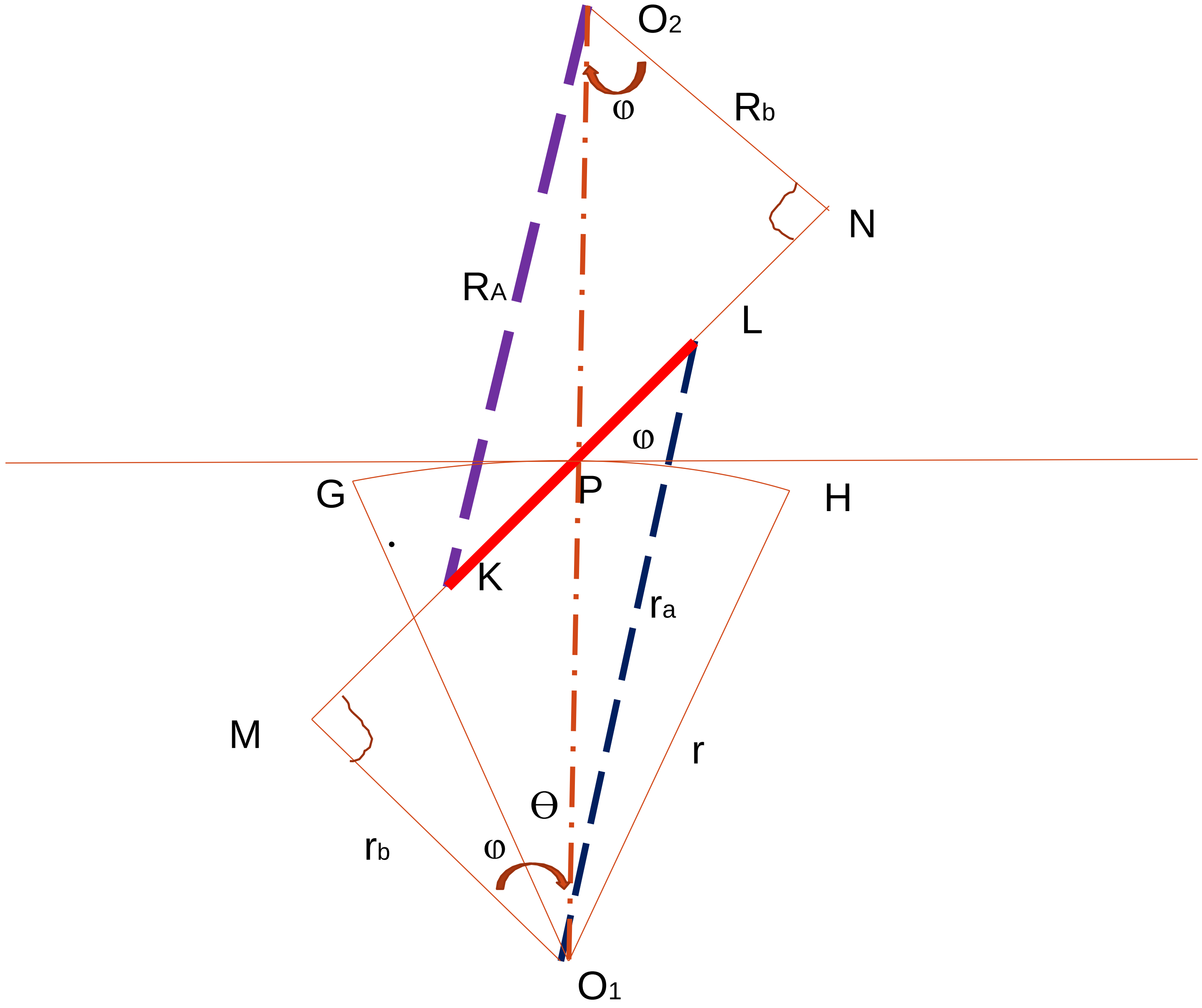
Hence the path of approach

$$KP = KN - PN$$

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$



PATH OF CONTACT



PATH OFCONTACT

- **radius of the base circle of pinion,**

$$O_1M = O_1P\cos\phi = r\cos\phi$$

- **radius of the base circle of wheel,**

$$O_2N = O_2P\cos\phi = R\cos\phi$$

- **Similarly from right angled triangle *O₁ML***

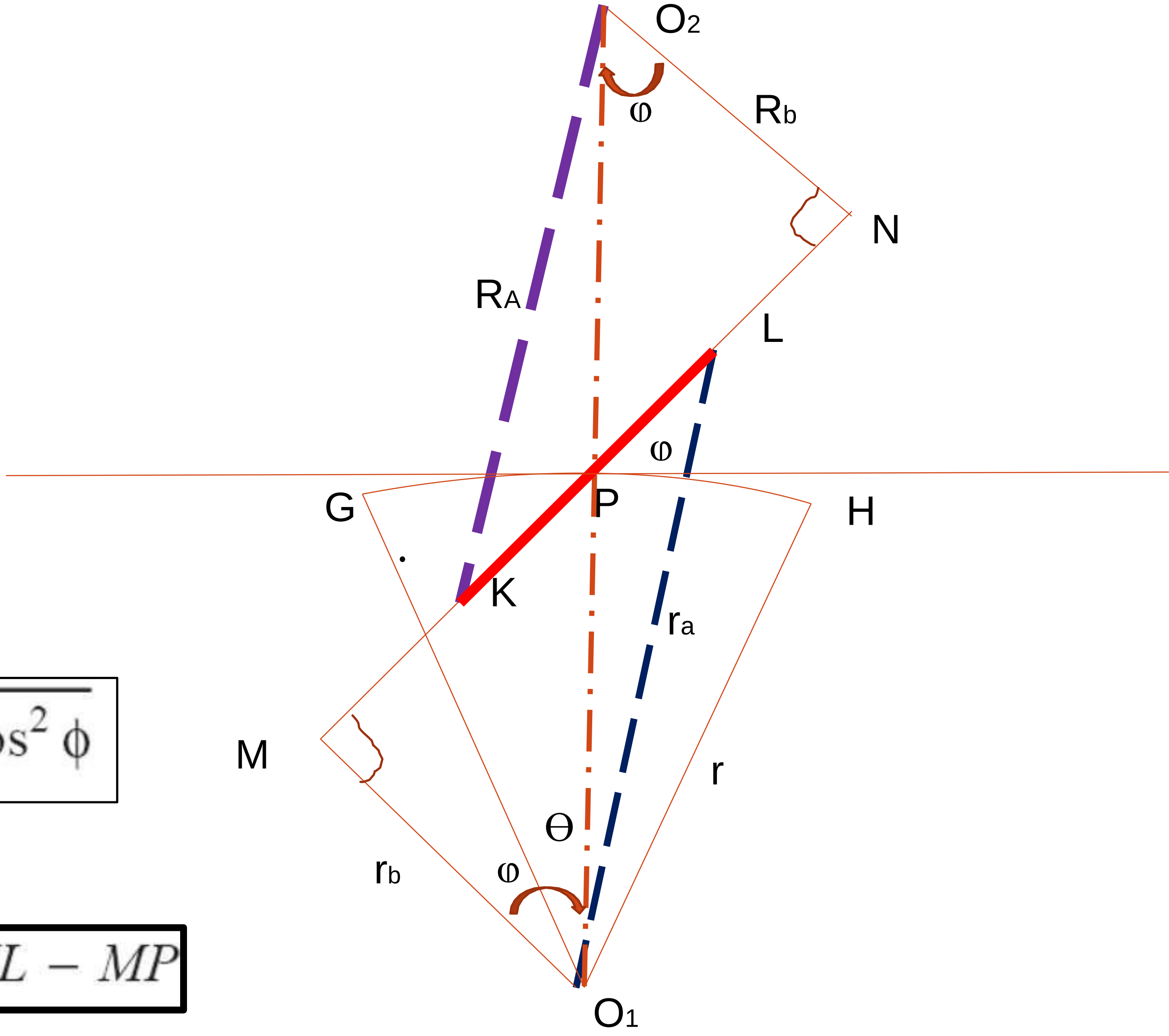
$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

<u>Hence the path of recess</u>

$$PL = ML - MP$$

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$



PATH OFCONTACT

path of approach

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

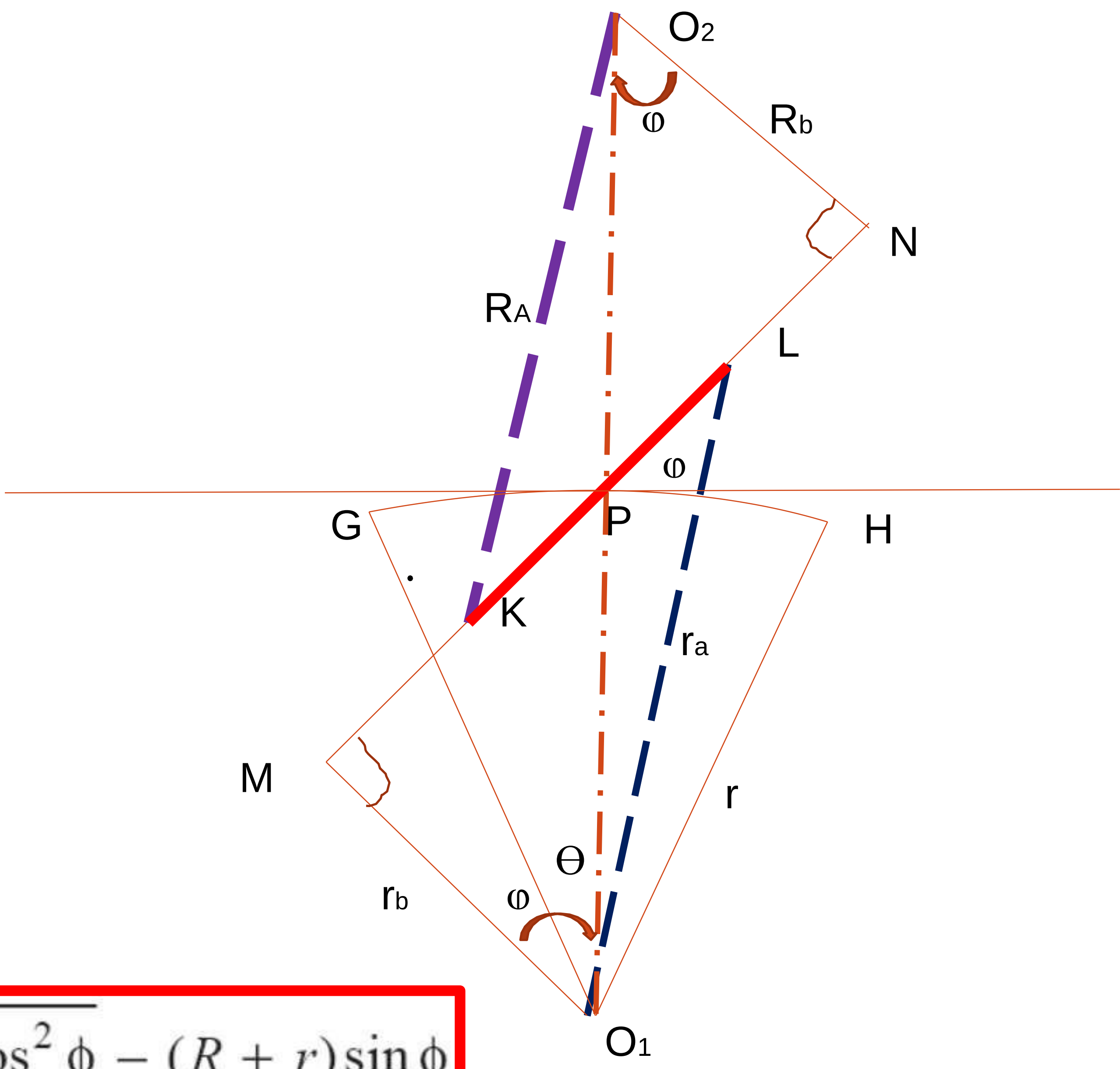
the path of recess

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

Length of the path of contact,

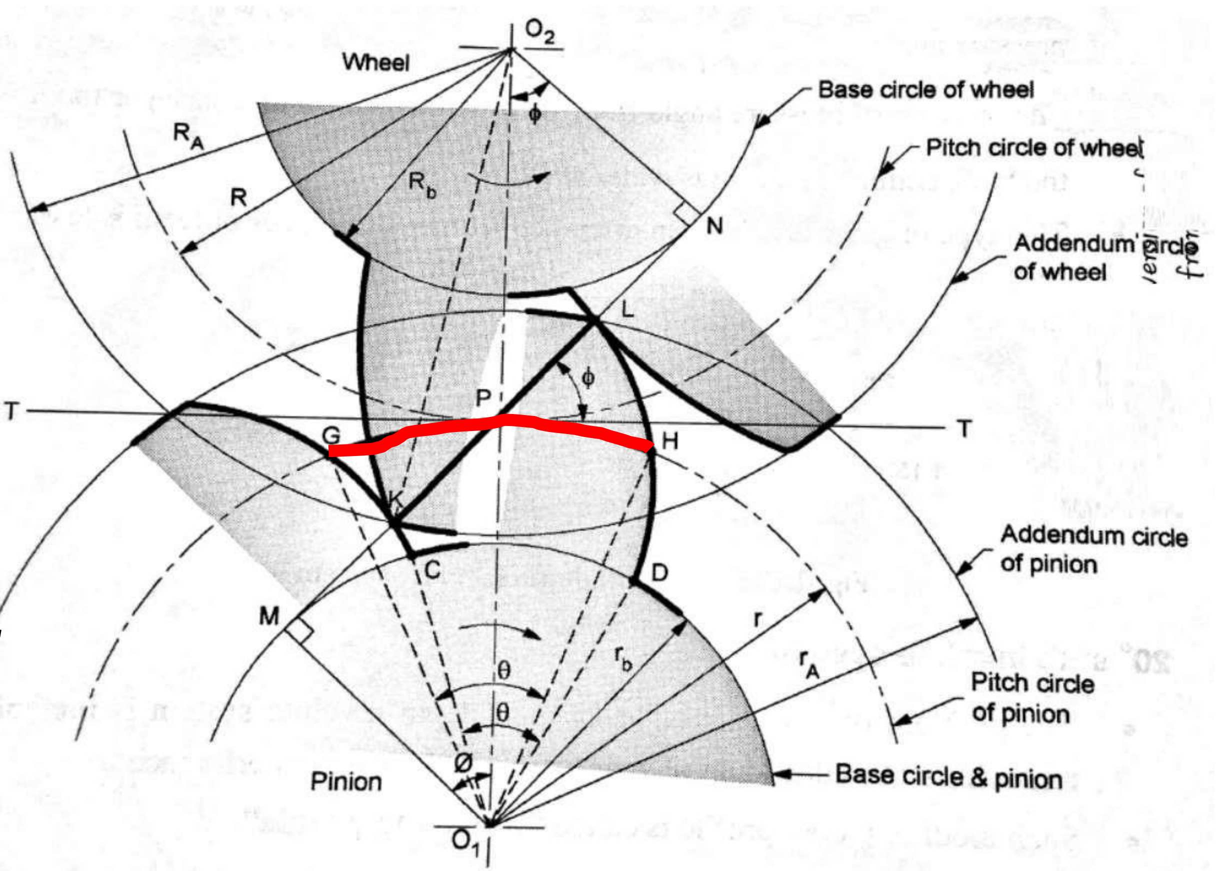
$$KL = KP + PL$$

$$KL = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} + \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$



ARC OF CONTACT

- **The arc of contact is *EPF* or *GPH*.**
- **Considering the arc of contact *GPH*, it is divided into two parts *i.e.* arc *GP* and arc**
- ***PH*.**
- **The arc *GP* is known as *arc of approach* and the arc *PH* is called *arc of recess*.**



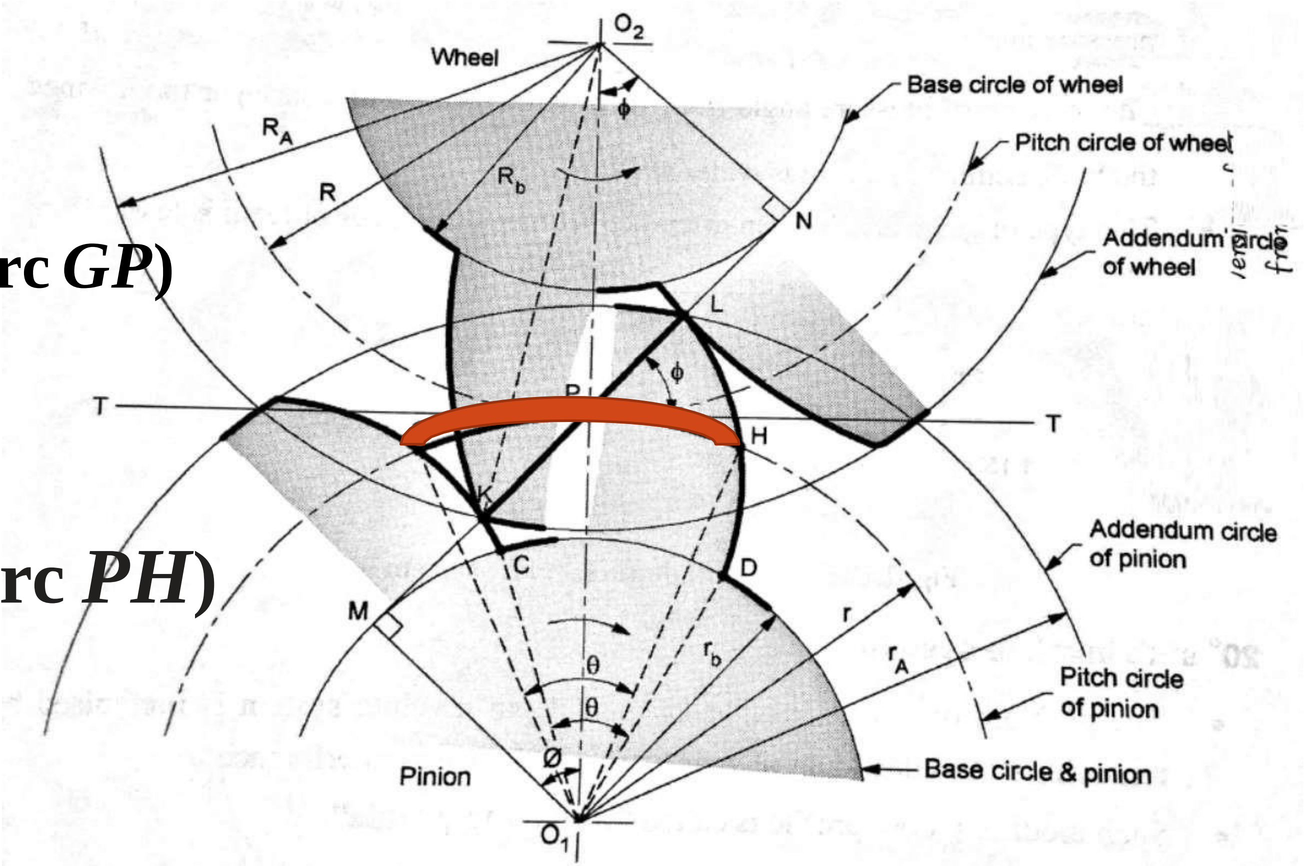
ARC OF CONTACT

- **the length of the arc of approach (arc *GP*)**

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

the length of the arc of recess (arc *PH*)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$



Since the length of the arc of contact *GPH* is equal to the sum of the length of arc of approach and arc of recess, therefore, Length of the arc of contact

$$\boxed{= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}}$$

$$\boxed{= \frac{\text{Length of path of contact}}{\cos\phi}}$$

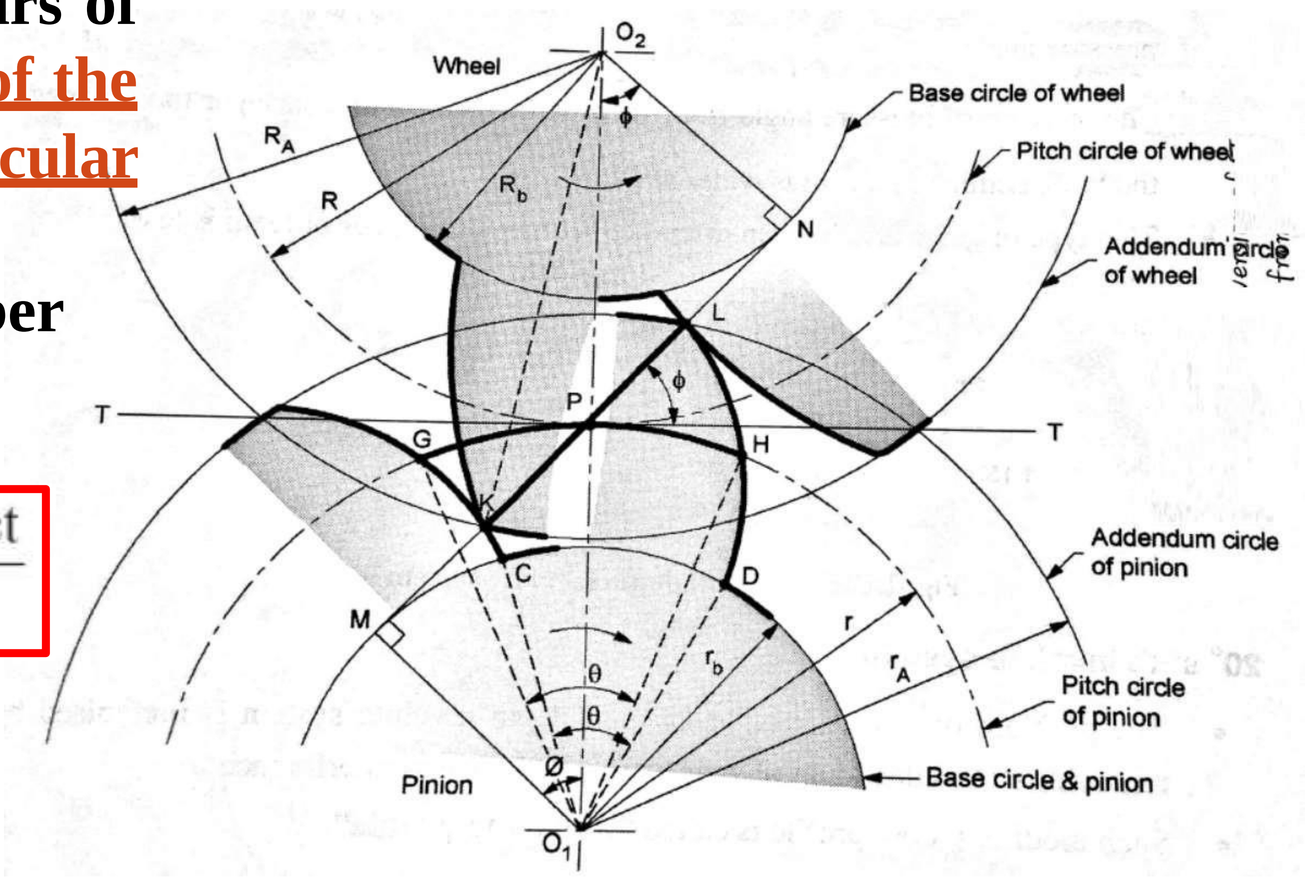
CONTACT RATIO (OR NUMBER OF PAIRS OF TEETH IN CONTACT)

- **The contact ratio or the number of pairs of teeth in contact is defined as the <u>ratio of the length of the arc of contact to the circular pitch.</u>**
- **Mathematically, Contact ratio or number of pairs of teeth in contact**

$$\boxed{\text{Contact ratio} = \frac{\text{Length of the arc of contact}}{p_c}}$$

where p_c = Circular pitch = πm, and

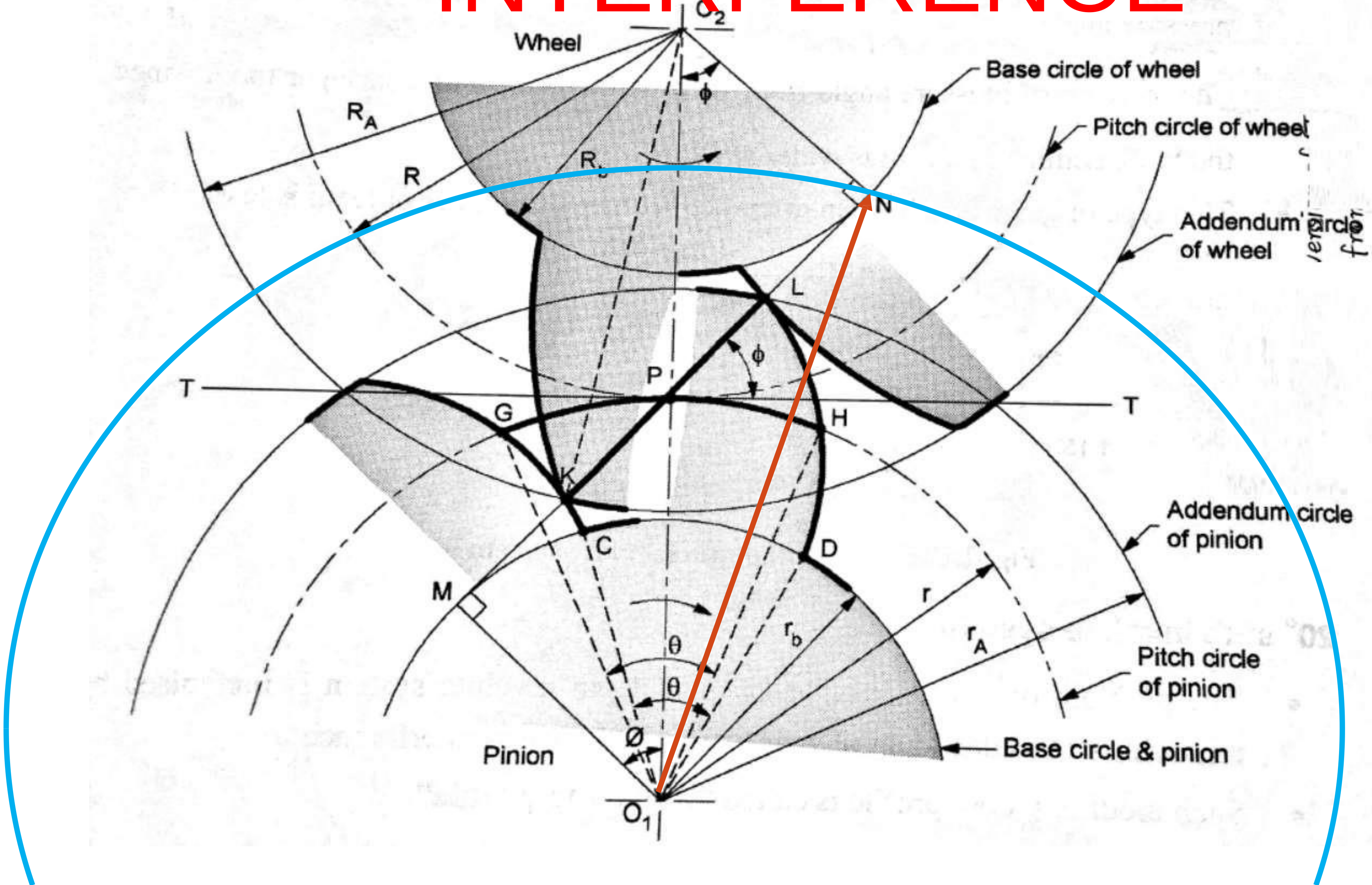
m = Module.



NUMERICAL

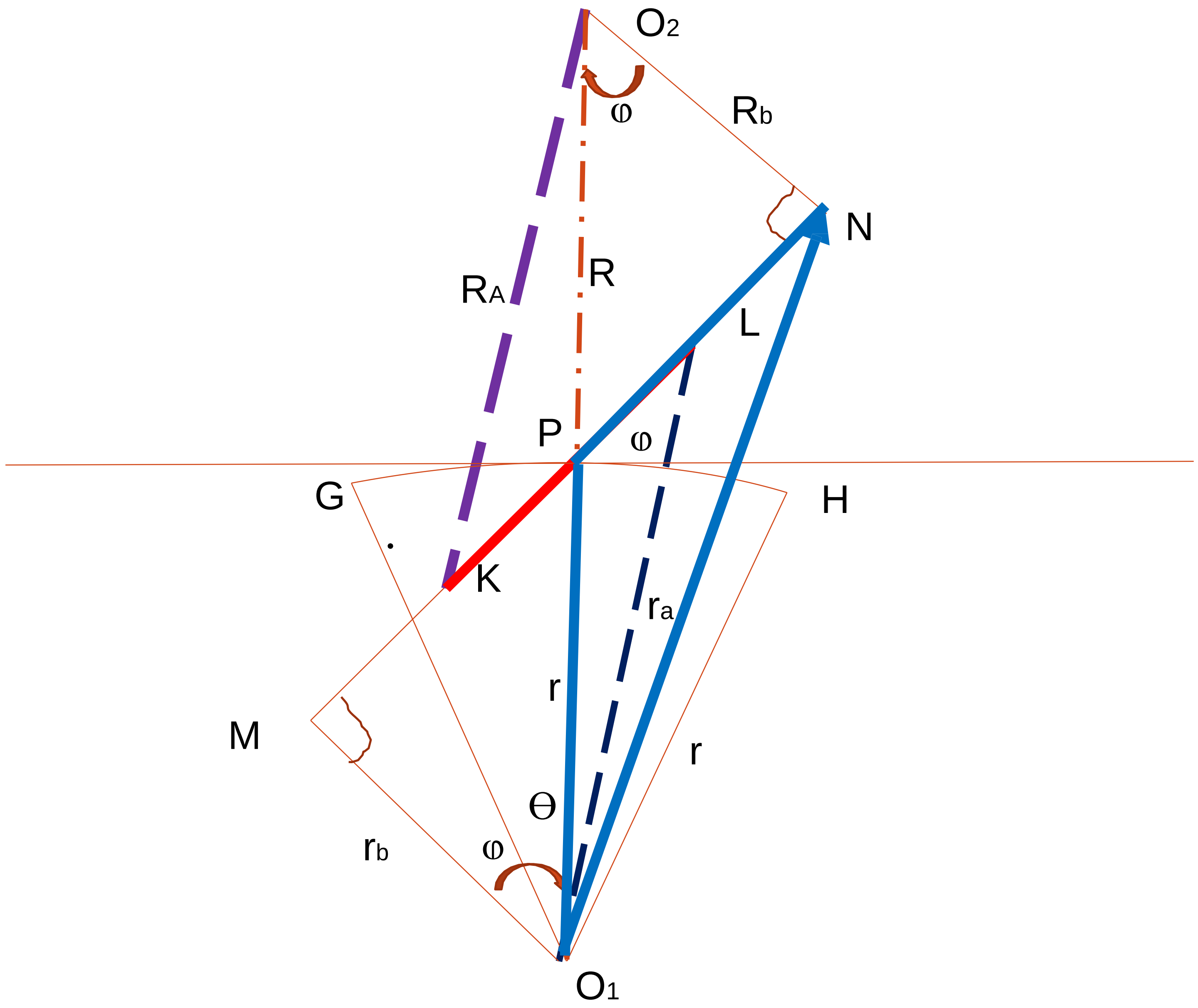
- *The number of teeth on each of the two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum.*
- Given : $T = t = 40$; φ= 20° ; m = 6 mm
- Adendum=6.12 mm

INTERFERENCE

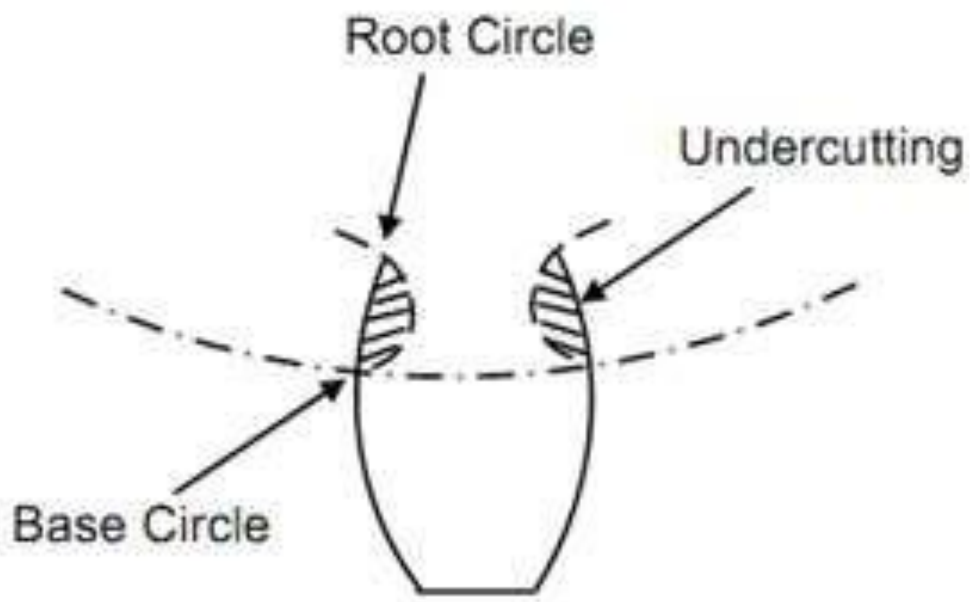


INTERFERENCE

- A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will move from L to N.
- When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel.
- The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel.
- This effect is known as ***interference,*** and occurs when the teeth are being cut. In brief, ***the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.***



O_2
φ
R_b
N
R_A
R
L
P
φ
G
H
K
r_a
r
M
r
θ
r_b
φ
O_1


Root Circle
Undercutting
Base Circle

INTERFERENCE

- The points M and N are called ***interference points.***
- Obviously, interference may be avoided if the path of contact does not extend beyond interference points.
- The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel is O_2M.
- we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth.
- In other words, ***interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.***

STANDARD PROPORTIONS OF GEAR SYSTEMS

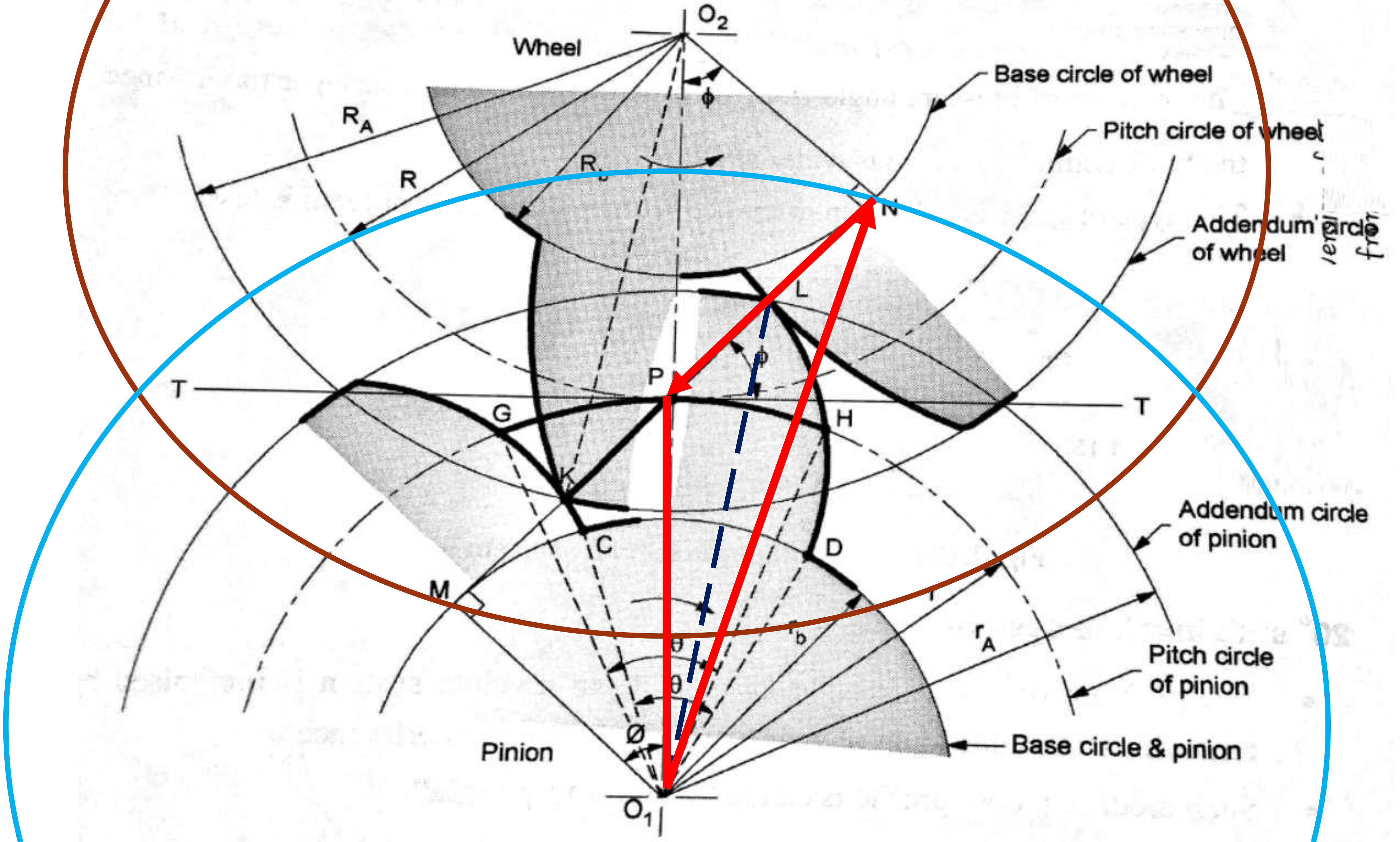
Standard proportions of gear systems.

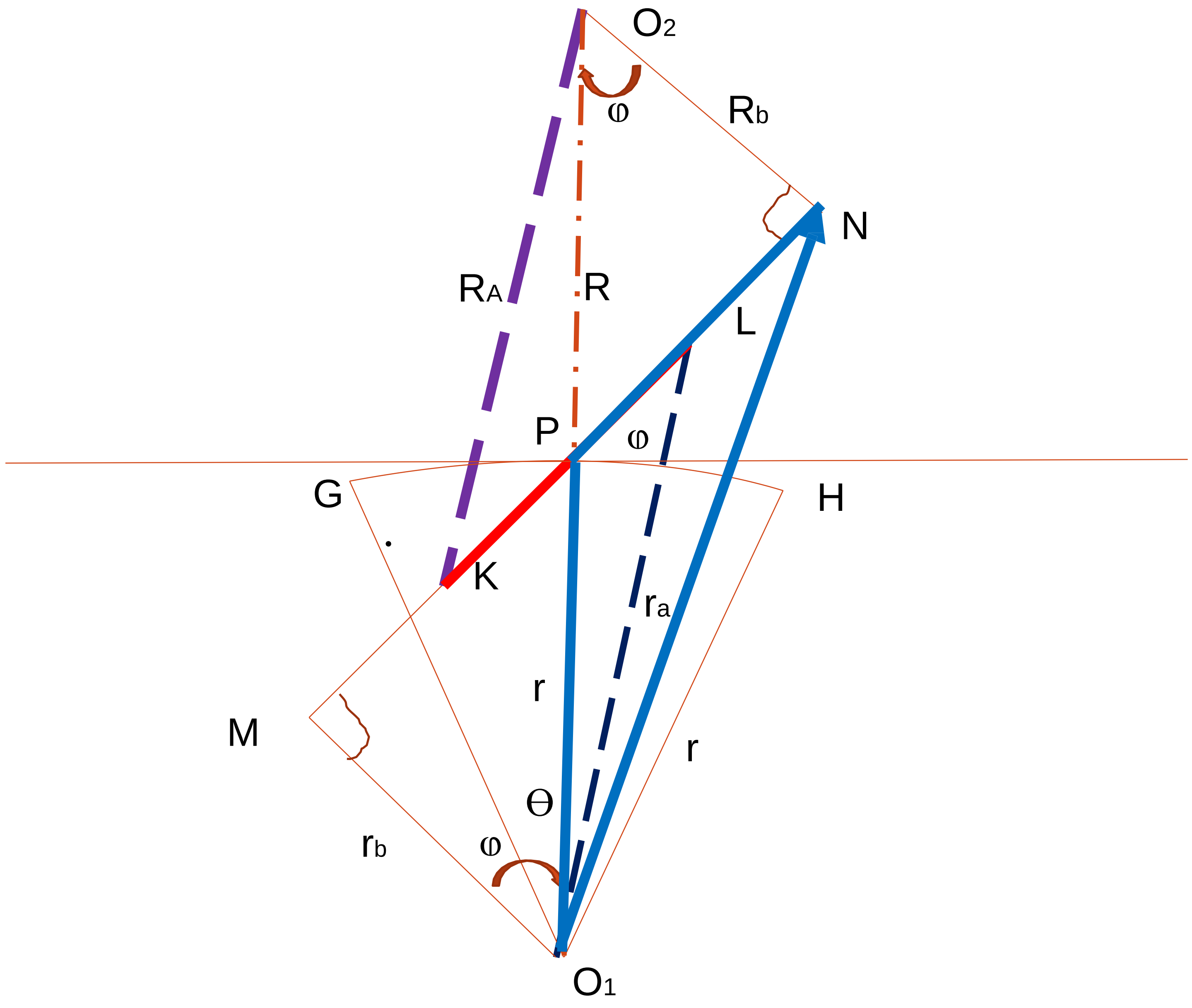
S. No.	*Particulars*	$14\frac{1}{2}°$ *composite or full depth involute system*	*20° full depth involute system*	*20° stub involute system*
1.	Addenddm	1 *m*	1 *m*	0.8 *m*
2.	Dedendum	1.25 *m*	1.25 *m*	1 *m*
3.	Working depth	2 *m*	2 *m*	1.60 *m*
4.	Minimum total depth	2.25 *m*	2.25 *m*	1.80 *m*
5.	Tooth thickness	1.5708 *m*	1.5708 *m*	1.5708 *m*
6.	Minimum clearance	0.25 *m*	0.25 *m*	0.2 *m*
7.	Fillet radius at root	0.4 *m*	0.4 *m*	0.4 *m*

Minimum number of teeth on the pinion

S. No.	*System of gear teeth*	*Minimum number of teeth on the pinion*
1.	$14\frac{1}{2}^{\circ}$ Composite	12
2.	$14\frac{1}{2}^{\circ}$ Full depth involute	32
3.	20° Full depth involute	18
4.	20° Stub involute	14

MINIMUM NUMBER OF TEETH ON THE PINION IN ORDER TO AVOID INTERFERENCE





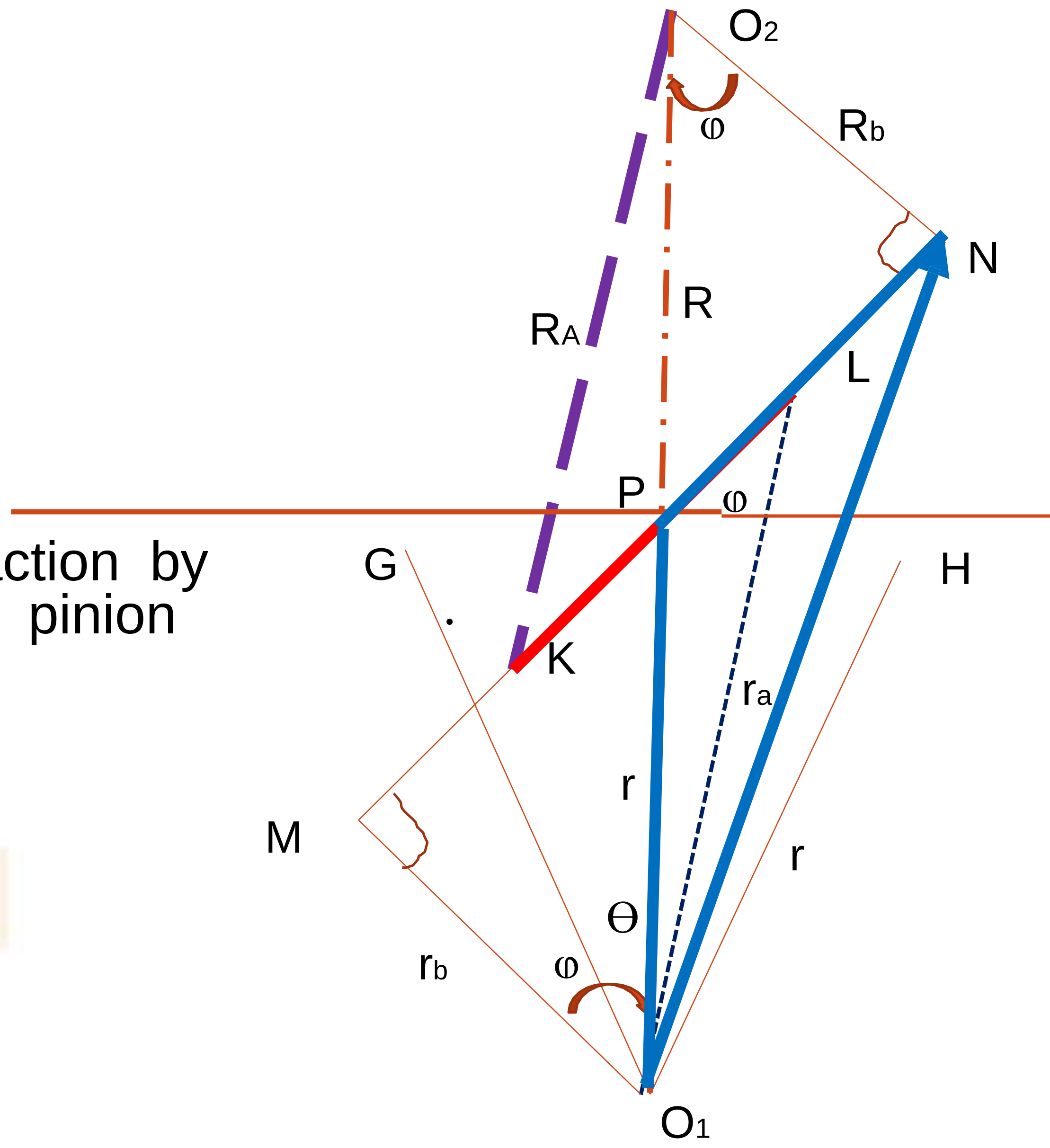
O2
Rb
φ
N
RA
R
L
P
φ
G
H
K
ra
r
M
r
Θ
rb
φ
O1

- t = Number of teeth on the pinion,,
- T = Number of teeth on the wheel,
- m = Module of the teeth,
- r = Pitch circle radius of pinion = $m.t$ / 2
- G = Gear ratio = T / t = R / r
- φ=Pressure angle or angle of obliquity.
- Let A_P.m = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in orderto avoid interference.
- From triangle O_1NP,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$...(\because\ PN = O_2P \sin\phi = R\sin\phi)$$



$$(O_1N)^2 = r^2 + R^2 \sin^2\phi + 2r.R\sin^2\phi$$

$$= r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$\therefore$ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$

We know that the addendum of the pinion

$$= O_1N - O_1P$$

$$\therefore \qquad A_P.m = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{m.t}{2} \qquad \ldots(\because O_1P = r = m.t/2)$$

$$A_P.m = \frac{m.t}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - \frac{m.t}{2}$$

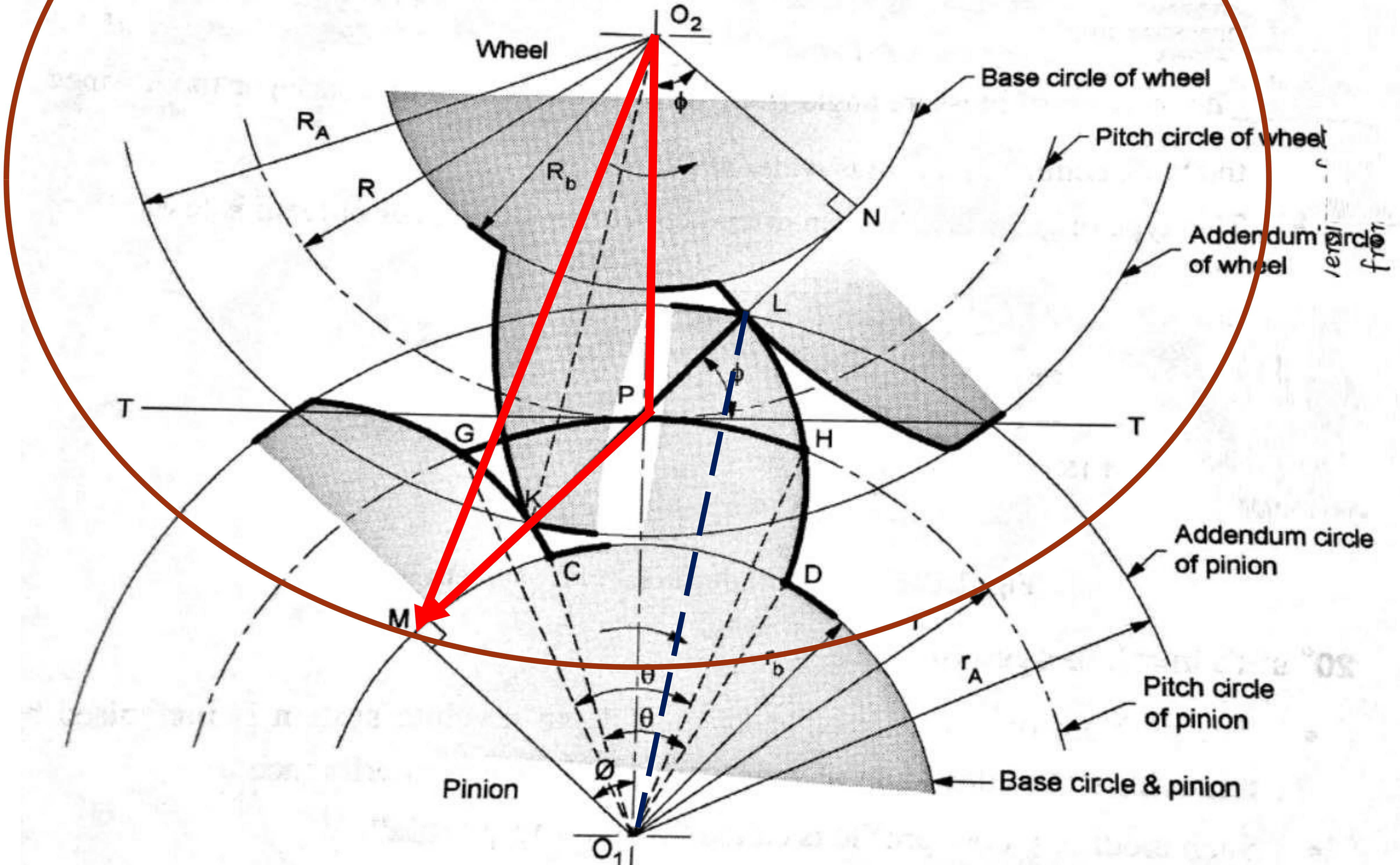
$$A_P.m = \frac{m.t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

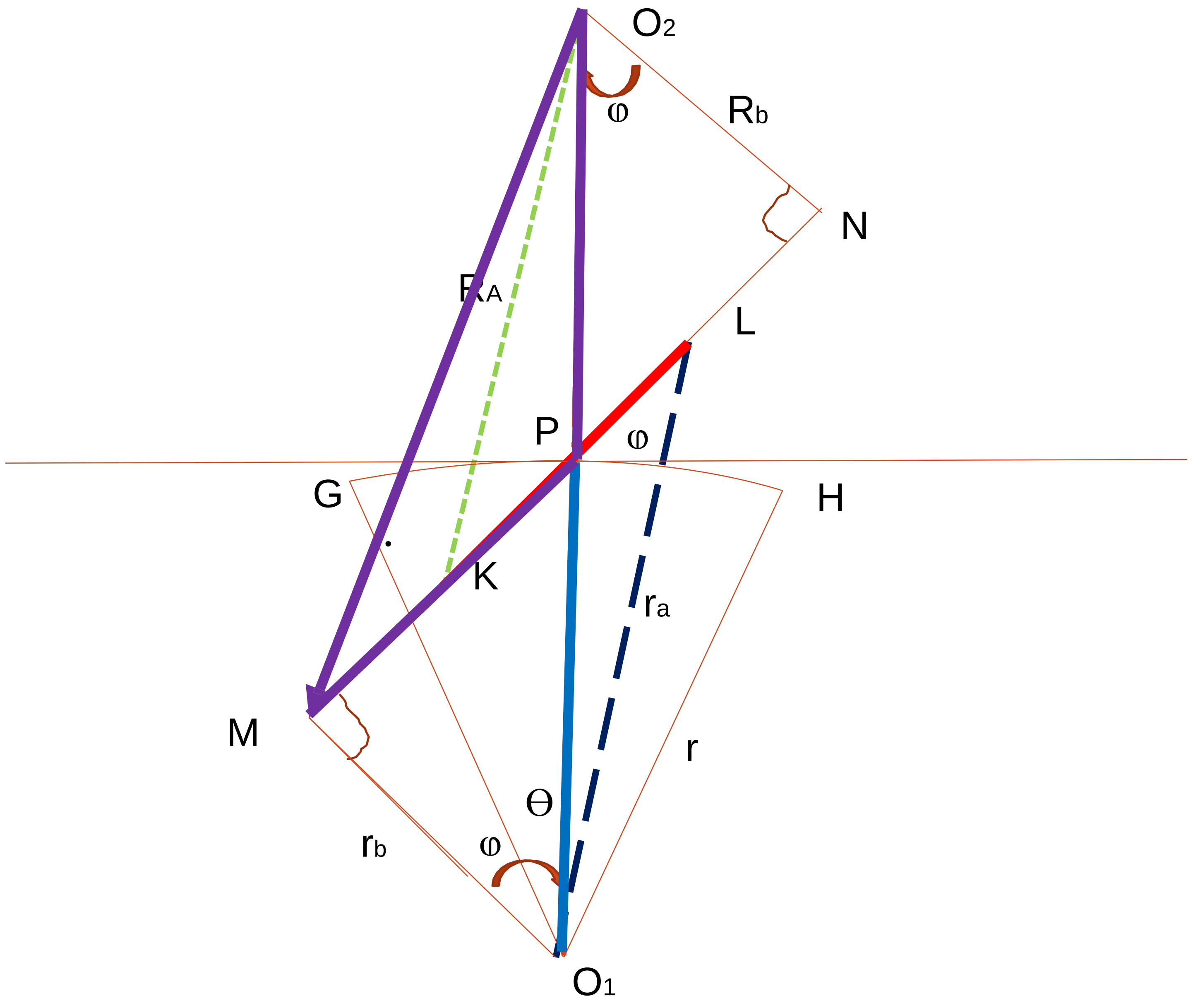
$$A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$\therefore \quad t = \frac{2A_P}{\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1+G(G+2)\sin^2\phi} - 1}$$

This equation gives the minimum number of teeth required on the pinion in order to avoid interference.

MINIMUM NUMBER OF TEETH ON THE WHEEL IN ORDER TO AVOID INTERFERENCE





O2
ϖ
Rb
N
RA
L
P
ϖ
G
H
K
ra
M
r
Θ
rb
ϖ
O1

- T = Minimum number of teeth required on the wheel in order to avoid interference, And
- $A_W.m$ = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should be multiplied.
- from triangle O_2MP

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2 \sin^2\phi - 2\,R.r \sin\phi \cos(90° + \phi)$$

$$\ldots(\because PM = O_1P \sin\phi = r)$$

$$T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

MINIMUM NUMBER OF TEETH TO AVOID INTERFERENCE

On pinion

$$\therefore \quad t = \frac{2A_P}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

On wheel

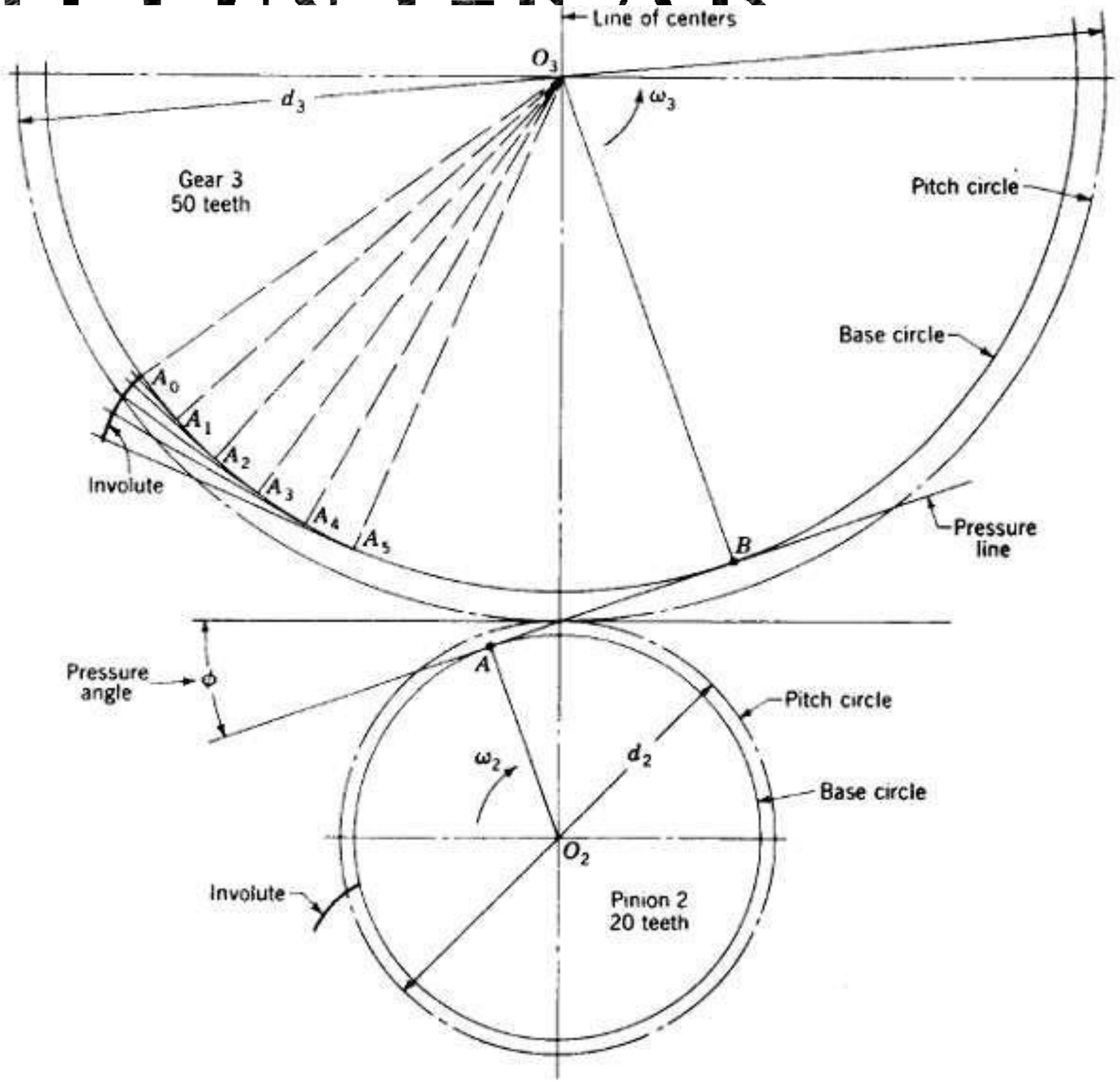
$$T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

NUMERICAL

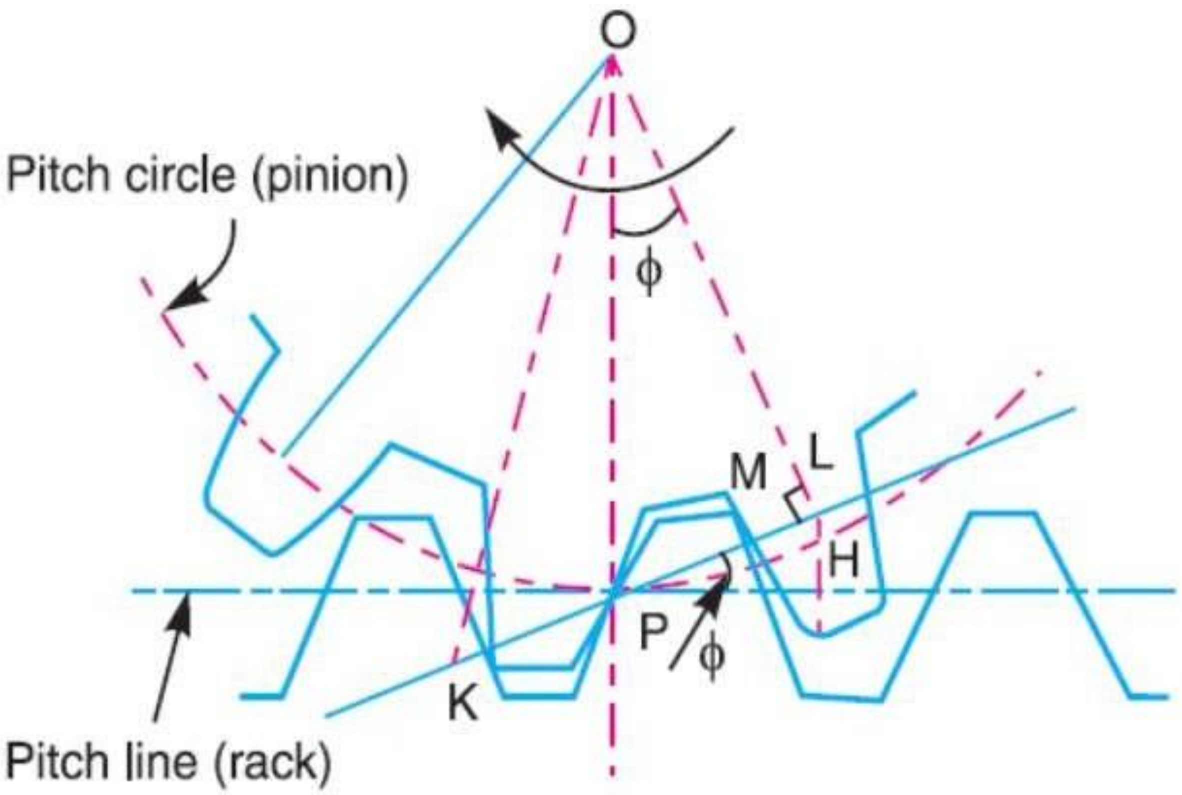
- *Determine the minimum number of teeth required on a pinion, in order to avoid interference which is to gear with,*
- ***1.*** *a wheel to give a gear ratio of 3 to 1 ; and* ***2.*** *an equal wheel.*
- *The pressure angle is 20° and a standard addendum of 1 module for the wheel may be assumed.*
- Given : $G = T / t = 3$; $\varphi = 20°$; $A_W = 1$ module
- **t=16**
- **t=13**

- *A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. When the gear ratio is 1.75, find in order that the interference is just avoided ;* ***1.*** *the addenda on pinion and gear wheel ;* ***2.*** *the length of path of contact ; and* ***3.*** *the maximum velocity of sliding of teeth on either side of the pitch point.*
- Given : $\varphi = 16°$; $m = 6$ mm ; $t = 16$; $N1 = 240$ r.p.m. or $\omega_1 = 2\pi \times 240/60 = 25.136$ rad/s ;
- $G = T/t = 1.75$ or $T = G.t = 1.75 \times 16 = 28$
- Addendum of pinion=10.76mm
- Addendum of wheel=4.56mm
- $R_A = R$ + Addendum of wheel = 84 + 10.76 = 94.76 mm
- $r_A = r$ + Addendum of pinion = 48 + 4.56 = 52.56 mm
- KP=26.45mm; PL=11.94mm;
- $KL = KP + PL = 26.45 + 11.94 = 38.39$ mm
- W_2=14.28rad/sec
- Sliding velocity during engagement= 1043mm/s
- Sliding velocity during disengagement= 471mm/s

FORMATION OF INVOLUTE TEETH ON GEAR



MINIMUM NUMBER OF TEETH ON A PINION FOR INVOLUTE RACK IN ORDER TO AVOID INTERFERENCE



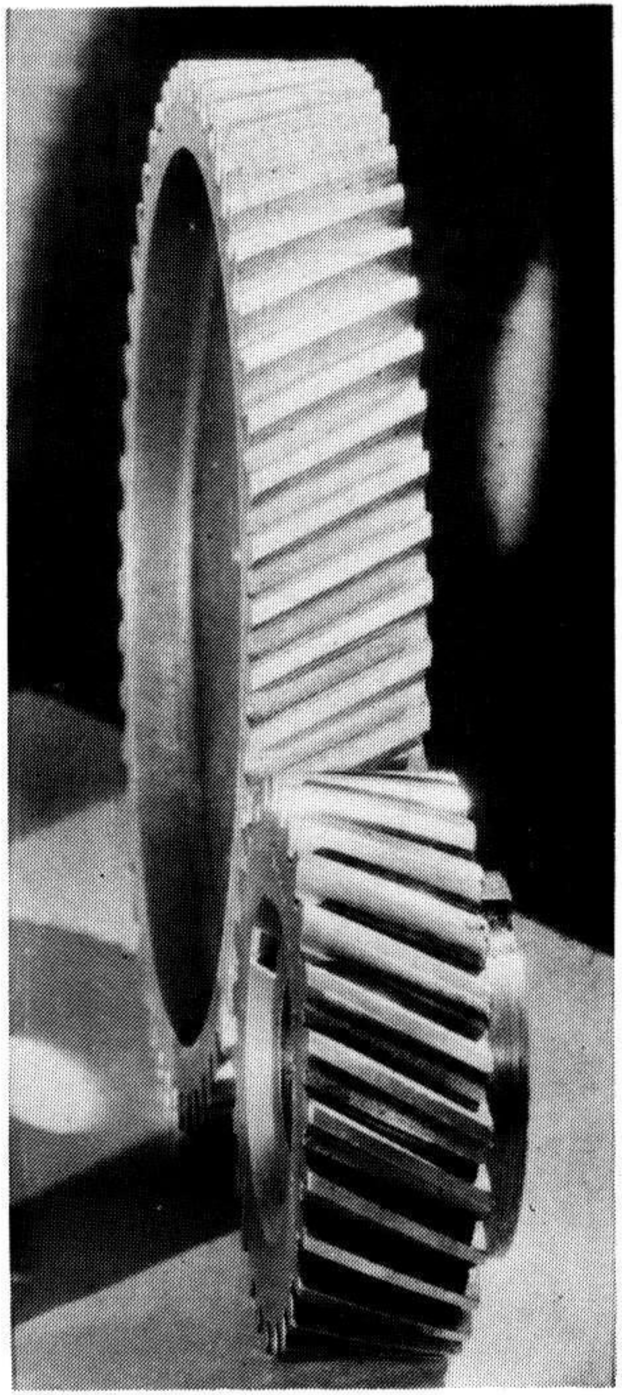
$$t = \frac{2\,A_R}{\sin^2 \phi}$$

HELICAL GEAR

- Helical gears can be used in a variety of applications since they can be mounted on either parallel or on 90° non-intersecting shafts.
- Helical gears offer additional benefits relative to spur gears:
  - Greater tooth strength due to the helical wrap around
  - Increased contact ratio due to the axial tooth overlap
  - Higher load carrying capacity than comparable sized spur gears.
- Smoother operating characteristics
- The close concentricity between the pitch diameter and outside diameter allow for smooth and quiet operation

SAMPLE APPLICATIONS

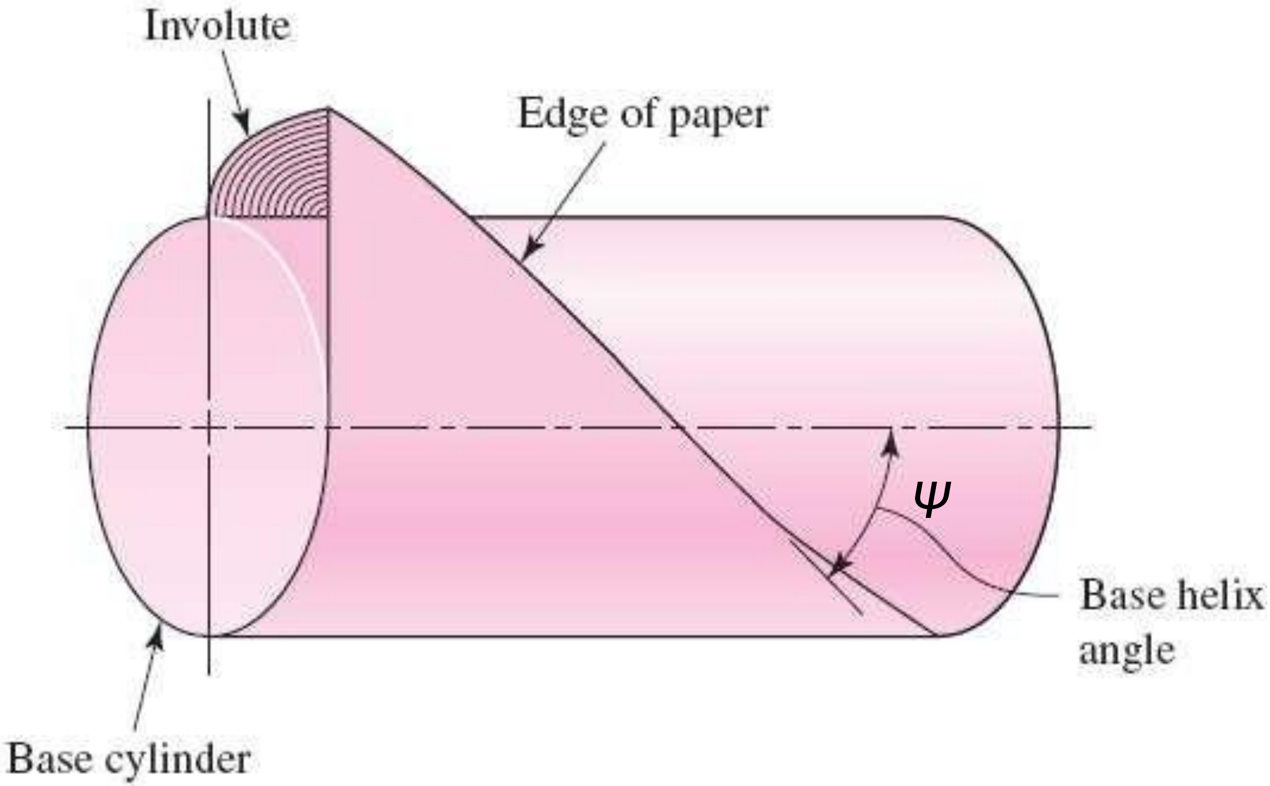
- Automobiles
- Presses
- Machine tools
- Material handling
- Feed drives
- Marine applications



HELICAL GEARS- KINEMATICS

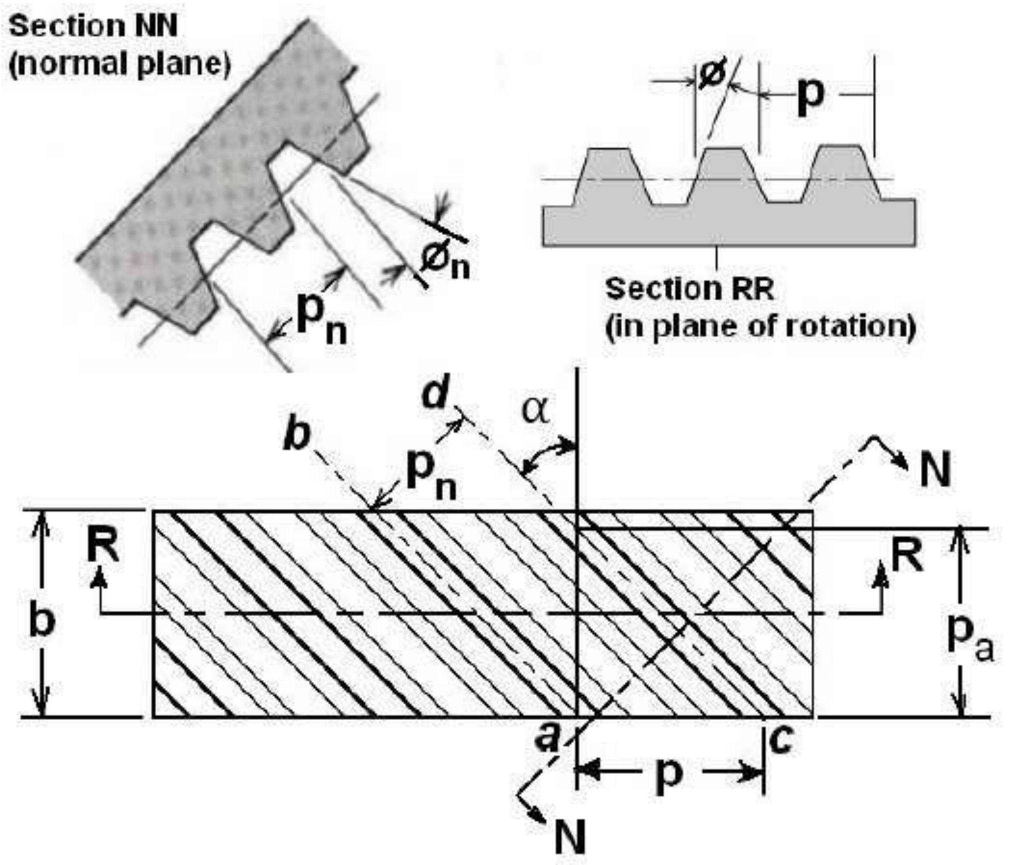
When two helical gears are engaged as in the, the helix angle has to be the same on each gear, but one gear must have a right-hand helix and the other a left-hand helix.

INVOLUTE HELICOID



The helix angle Ψ, is always measured on the cylindrical pitch surface Ψ value is not standardized. It ranges between 15° and 45°. Commonly used values are 15, 23, 30 or 45°. Lower values give less end thrust. Higher values result in smoother operation and more end thrust. Above 45° is not recommended.

TERMINOLOGY OF HELICAL GEAR



1. Helix angle: It is the angle at which teeth are inclined to the axis of gear. It is also known as spiral angle.

2.Normal pitch. It is the distance between similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth. It is denoted by Pn.

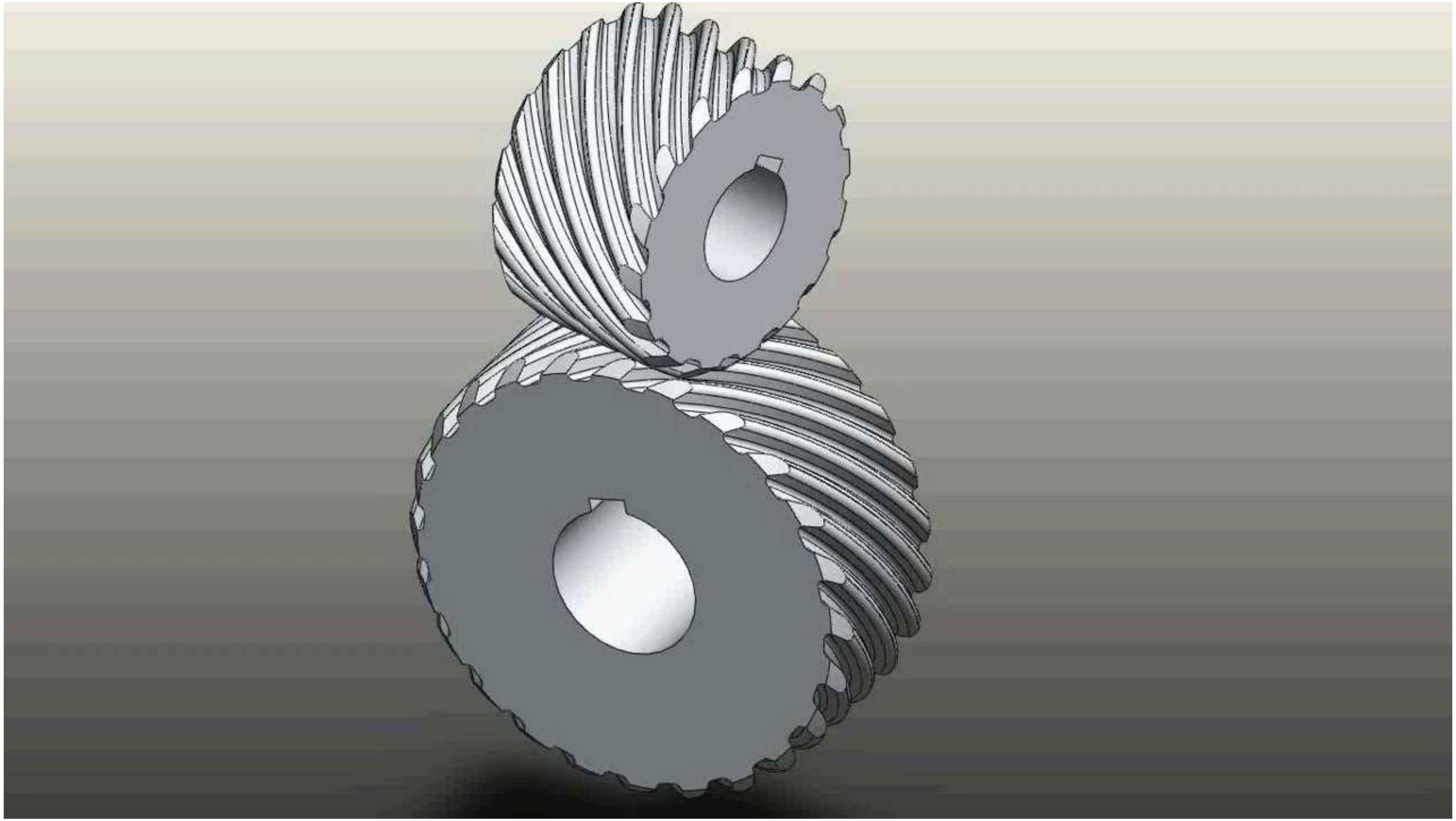
3.Transverse Pitch. It is the distance measured parallel to the axis, between similar faces of adjacent teeth denoted by P.

If ɑ is the helix angle, then circular pitch,

Φ_n normal pressure angle

SPIRAL GEARS

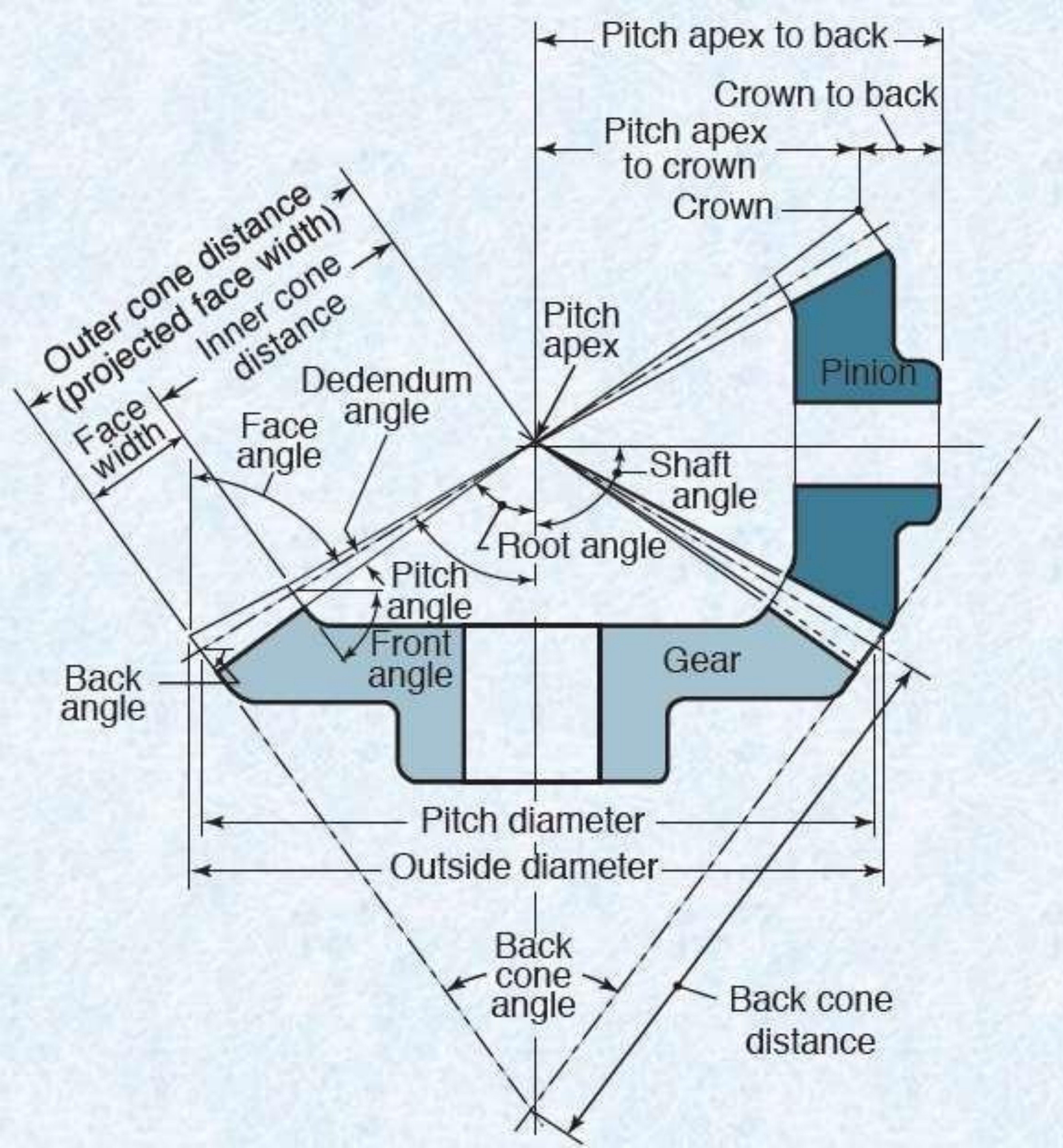
- spiral gears (also known as **skew gears** or **screw gears**) are used to connect and transmit motion between two non-parallel and non-intersecting shafts.
- The pitch surfaces of the spiral gears are cylindrical and the teeth have point contact.
- These gears are only suitable for transmitting small power.
- helical gears, connected on parallel shafts, are of opposite hand. But spiral gears may be of the same hand or of opposite hand.



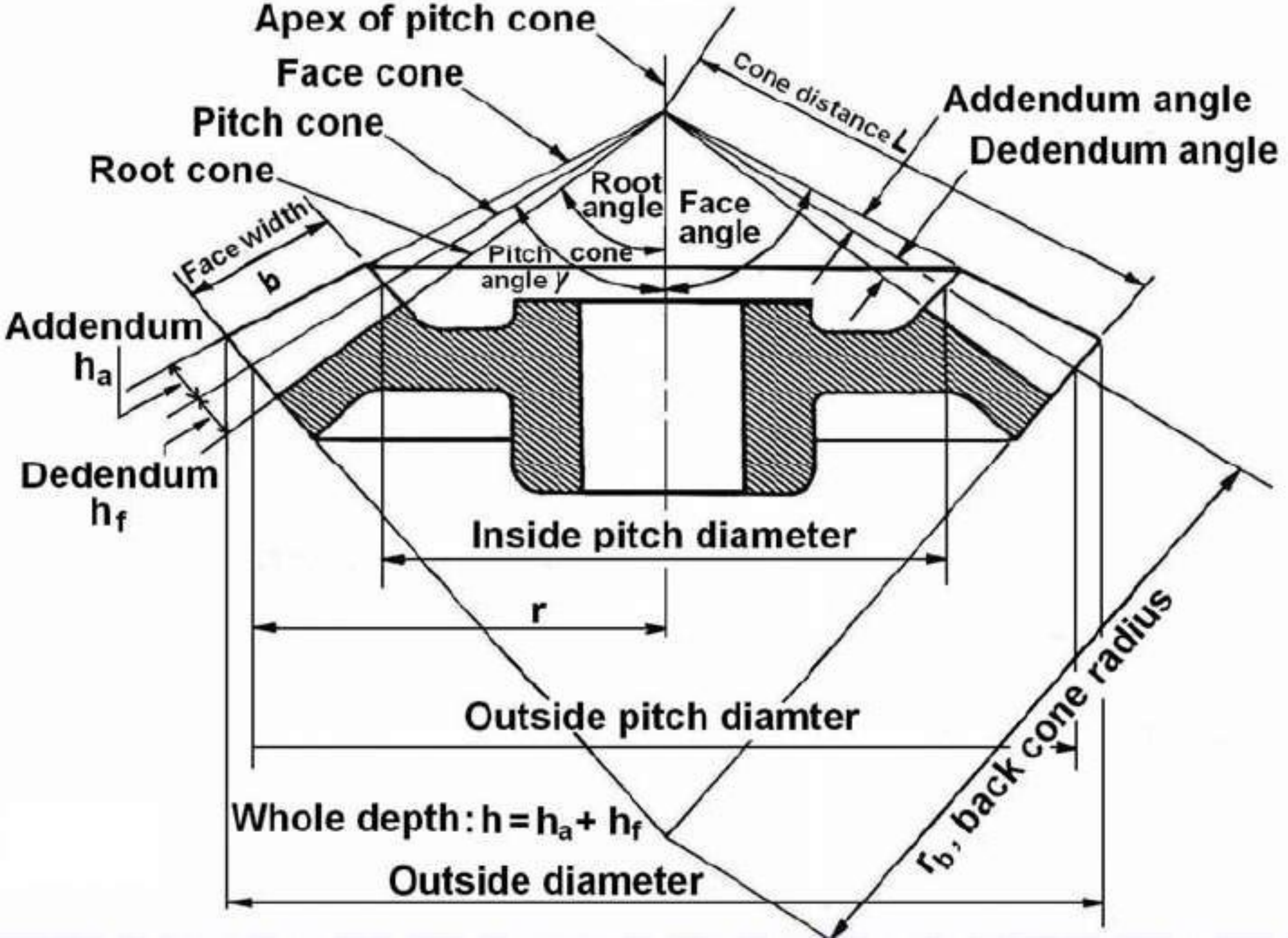
BEVEL GEARS




STRAIGHT BEVEL GEARS


Pitch apex to back
Crown to back
Pitch apex to crown
Crown
Outer cone distance (projected face width)
Inner cone distance
Pitch apex
Dedendum angle
Face angle
Face width
Pinion
Shaft angle
Root angle
Pitch angle
Front angle
Gear
Back angle
Pitch diameter
Outside diameter
Back cone angle
Back cone distance

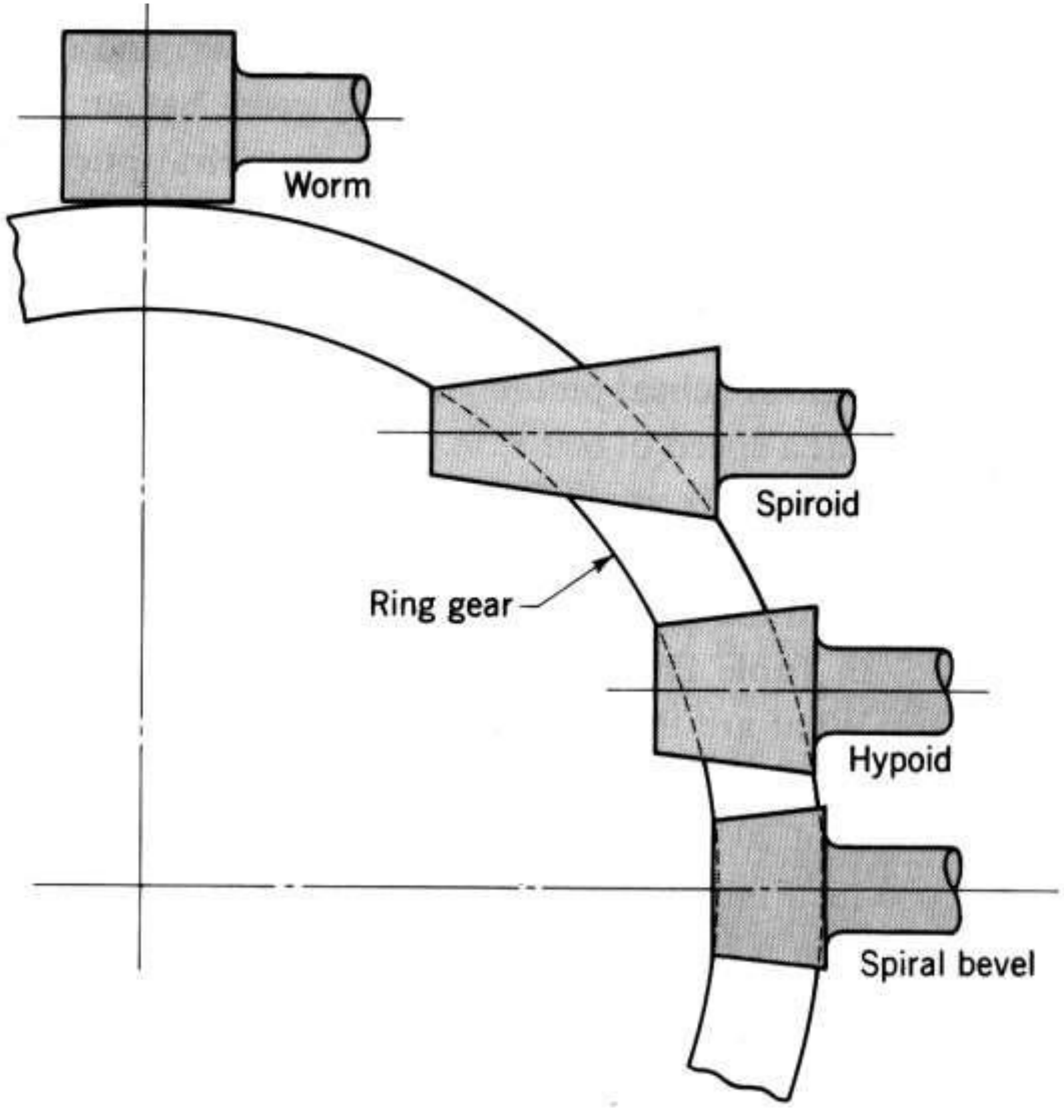
BEVEL GEARS


Apex of pitch cone
Face cone
Pitch cone
Root cone
Cone distance L
Addendum angle
Dedendum angle
Root angle
Face angle
Pitch cone angle γ
Face width b
Addendum h_a
Dedendum h_f
Inside pitch diameter
r
Outside pitch diamter
Whole depth: $h = h_a + h_f$
Outside diameter
r_b, back cone radius

EFFICIENCY OF GEARS

No	Type	Normal Ratio Range	Efficiency Range
1	Spur	1:1 to 6:1	94-98%
2	Straight Bevel	3:2 to 5:1	93-97%
3	Spiral Bevel	3:2 to 4:1	95-99%
4	Worm	5:1 to 75:1	50-90%
5	Hypoid	10:1 to 200:1	80-95%
6	Helical	3:2 to 10:1	94-98%
7	Cycloid	10:1 to 100:1	75% to 85%

BEVEL, HYPOID, SPIROID, WORM



GEAR TRAINS

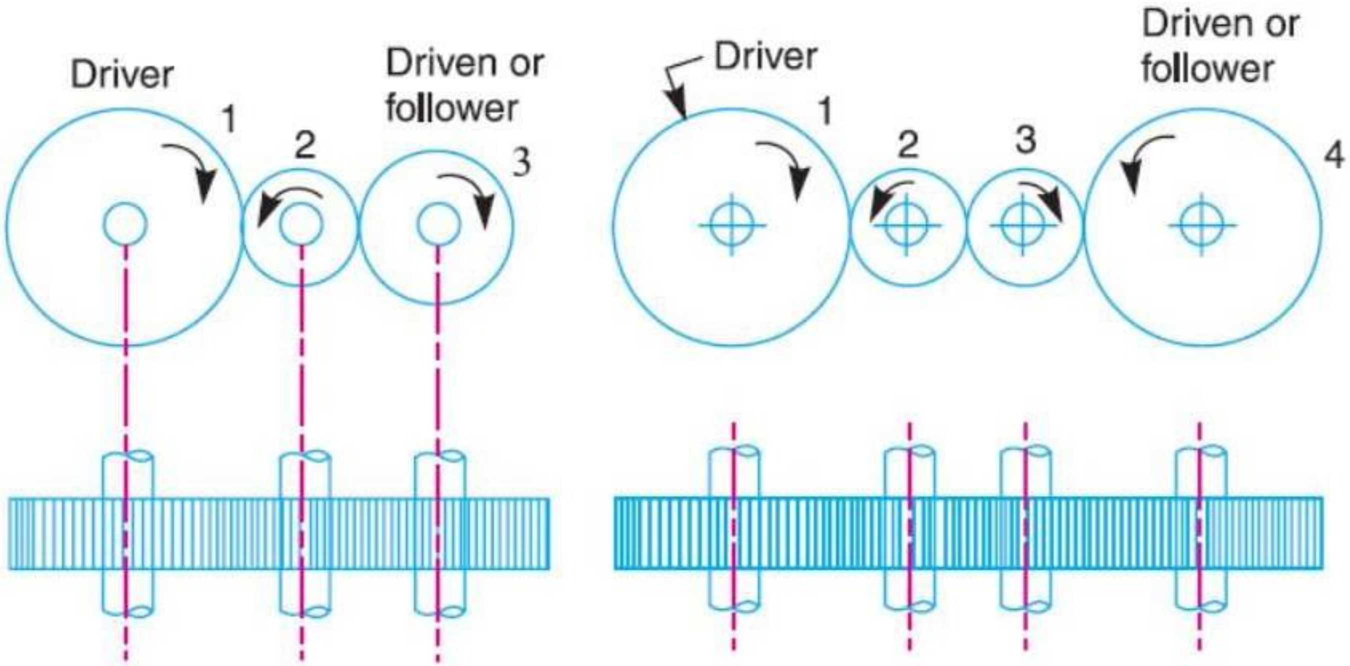
- Two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train*** or ***train of toothed wheels.***
  - **Depends on velocity ratio required and relative position of axes of the shafts.**
- A gear train may consist of spur, bevel or spiral gears.

❑ **Types of Gear Trains**

- 1. Simple gear train,
- 2. Compound gear train,
- 3. Reverted gear train, and
- 4. Epicyclic gear train.

SIMPLE GEAR TRAIN

- When there is only one gear on each shaft, it is known as ***simple gear train.***
- The gears are represented by their pitch circles.
- Gear 1 drives the gear 2, therefore gear 1 is called the ***driver*** and the gear 2 is called the ***driven*** or ***follower.***
- It may be noted that the motion of the driven gear is opposite to the motion of driving gear.



- **Speed ratio (or velocity ratio)** of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth,

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

- Ratio of the speed of the driven or follower to the speed of the driver is known as train value of the **train value.**

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

- speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears.

COMPOUND GEAR TRAIN

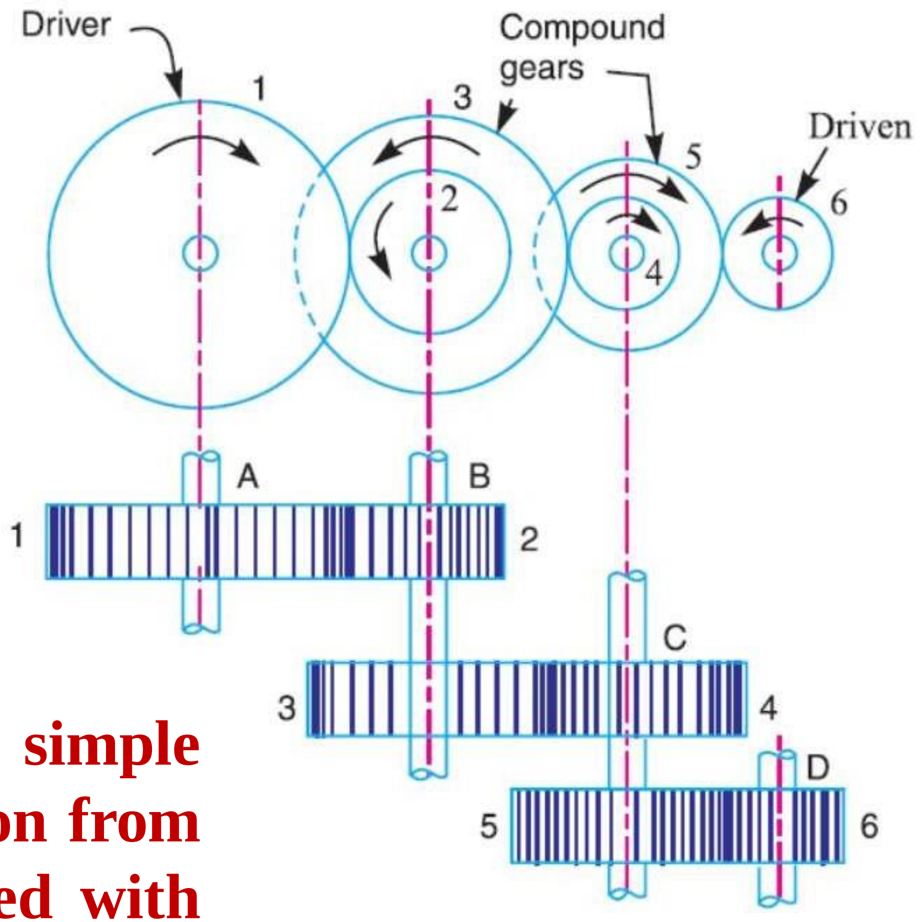
- When there are more than one gear on a shaft, it is called a ***compound train of gear***.

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

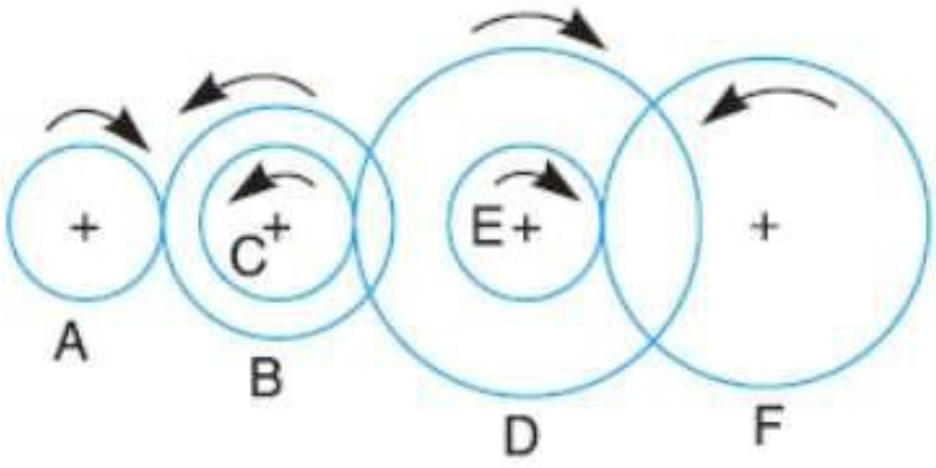
$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$



The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.

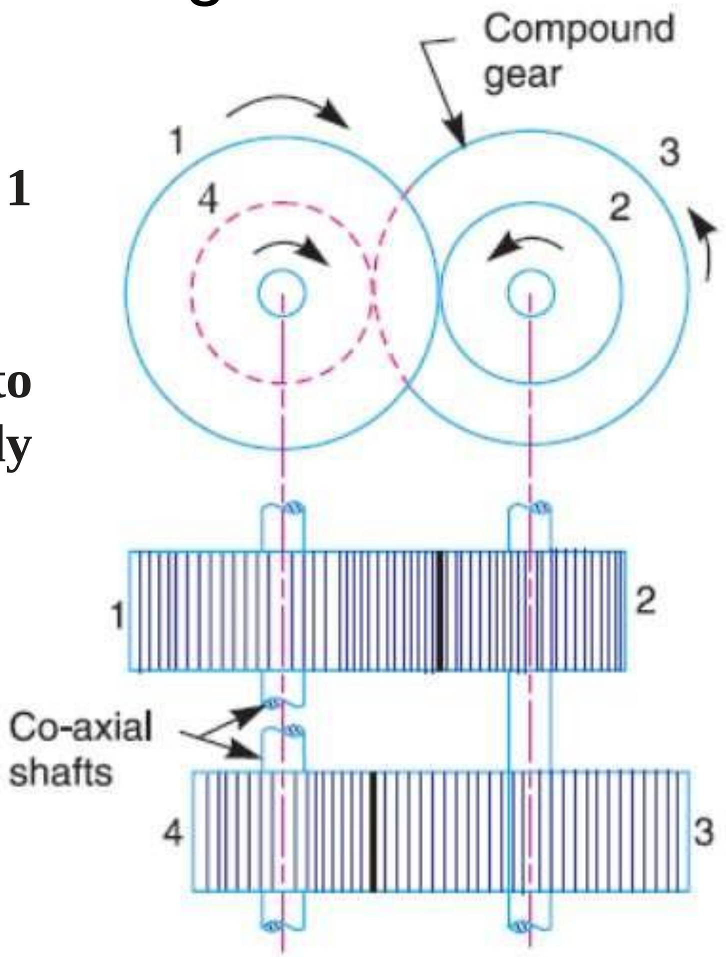
- *The gearing of a machine tool is shown in Fig.* 13.3. *The motor shaft is connected to gear A and rotates at* 975 *r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F ? The number of teeth on each gear are as given below :*

Gear	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth	*20*	*50*	*25*	*75*	*26*	*65*



REVERTED GEAR TRAIN

- When the axes of the first gear (*i.e.* first driver) and the last gear (*i.e.* last driven or follower) are co-axial, then the gear train is known as ***reverted gear* train** as shown i



Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$r1 + r2 = r3 + r4$ ...

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

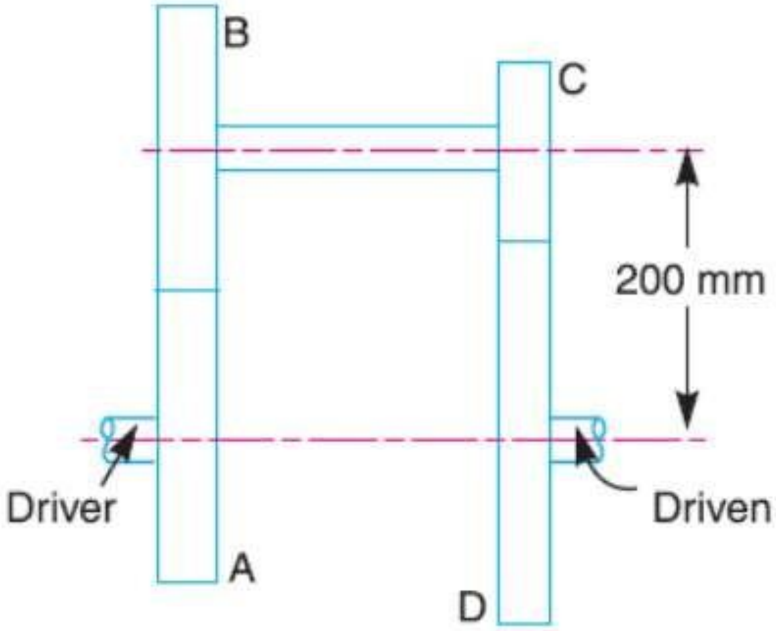
$T1 + T2 = T3 + T4$

- **Used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks**

(where minute and hour hand shafts are co axial)

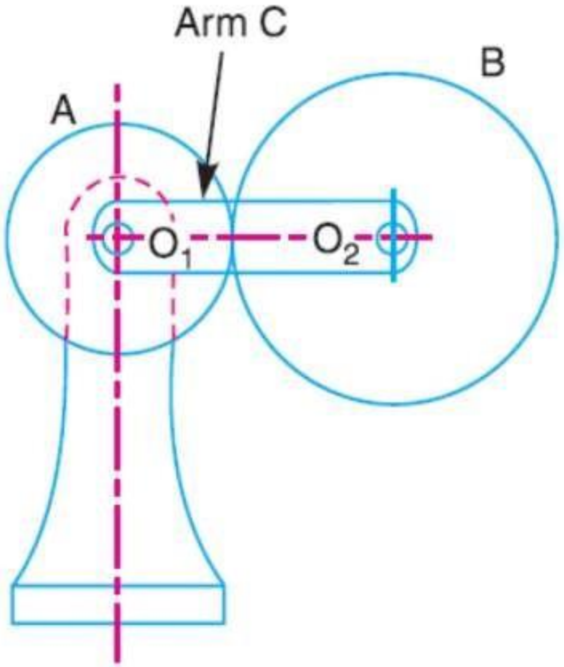
NUMERICAL

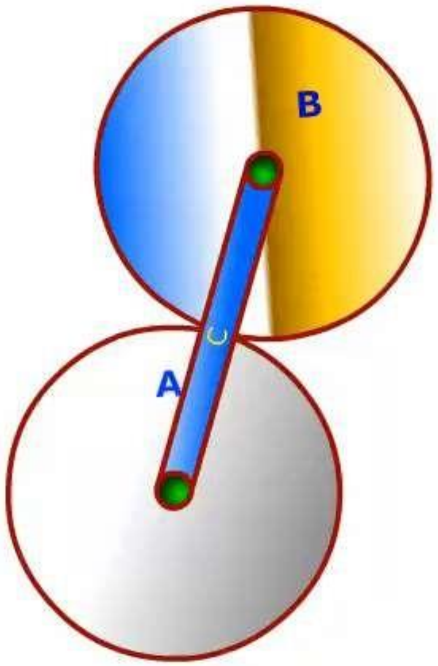
- *The speed ratio of the reverted gear train, as shown in Fig. 13.5, is to be 12. The module pitch of gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.*



EPICYCLIC GEAR TRAIN

- In epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.
- the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** (***epi***. means upon and ***cyclic*** means around). The epicyclic gear trains may be ***simple*** or ***compound***.
- **The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.**
- **The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.**





Simulation 2

Two degree of freedom

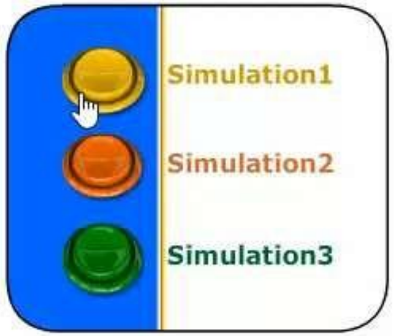
Input : No. of rotation

A = +1

C = +1

Output :

B = +1



VELOCITY RATIO IN EPICYCLIC GEAR TRAIN

METHOD 1: TABULAR FORM

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	$+1$	$-\dfrac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through $+x$ revolutions	0	$+x$	$-x \times \dfrac{T_A}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \dfrac{T_A}{T_B}$

VELOCITY RATIO IN EPICYCLIC GEAR TRAIN

METHOD 2: ALGEBRAIC FORM

- Let the arm C be fixed in an epicyclic gear train as shown in Fig.. Therefore speed of the
- gear A relative to the arm $C = N_A - N_C$
- and speed of the gear B relative to the arm C, $= N_B - N_C$
- Since the gears A and B are meshing directly, therefore they will revolve in ***opposite*** directions.

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

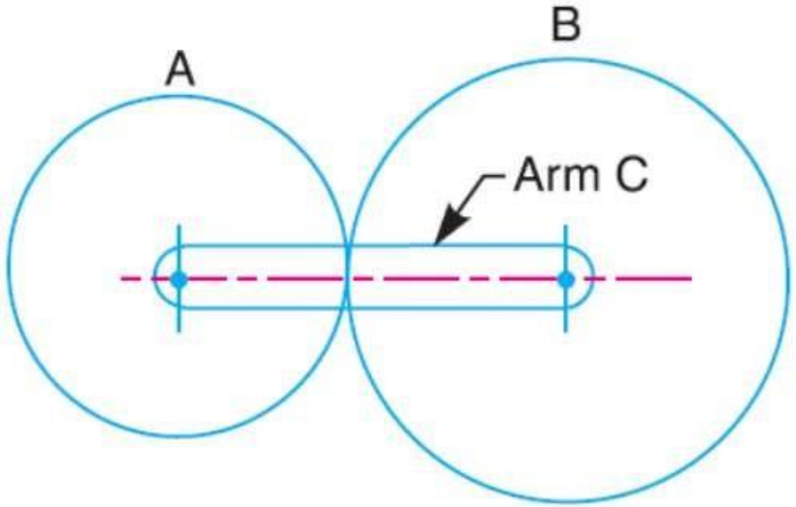
Since the arm C is fixed, therefore its speed, $N_C = 0$.

$$\therefore \qquad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

- *In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?*



Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

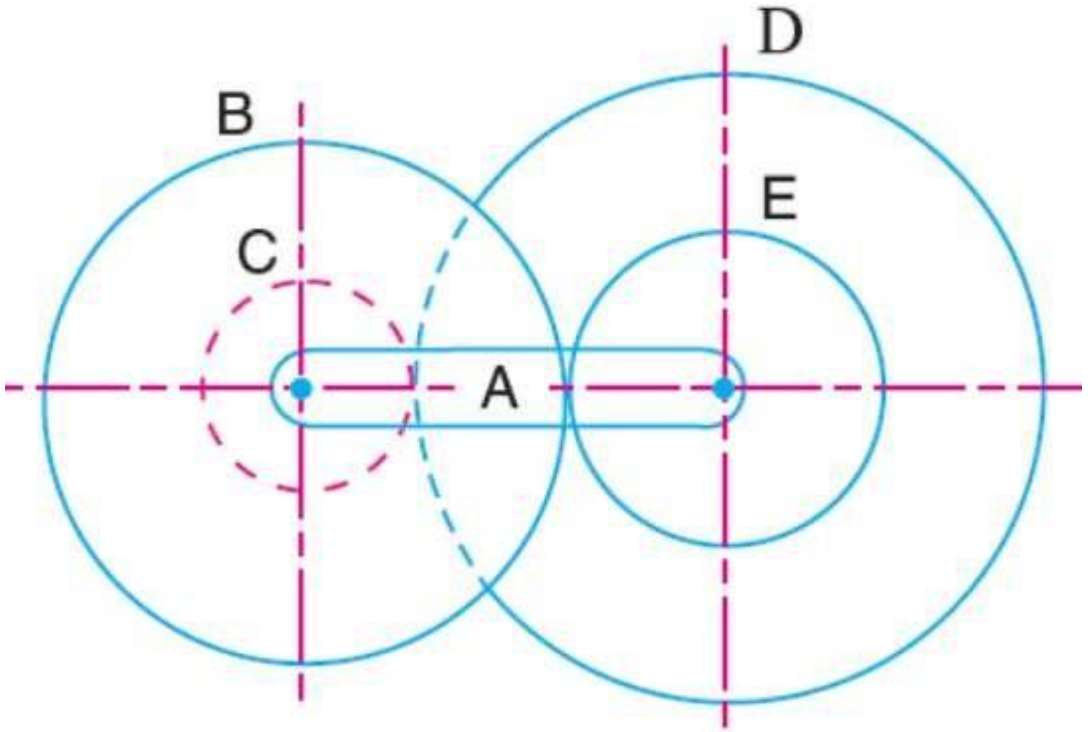
Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	$+1$	$-\dfrac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \dfrac{T_A}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \dfrac{T_A}{T_B}$

- ***Speed of gear B when gear A is fixed*** 270 r.p.m. (anticlockwise)
- ***Speed of gear B whe n gear A makes 300 r.p.m. clockwise*** = 510 r.p.m. (anticlock)

- *In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.*
- $T_B = 75$; $T_C = 30$; $T_D = 90$; $N_A = 100$ r.p.m. (clockwise)

$T_B + T_E = T_C + T_D$

$T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$

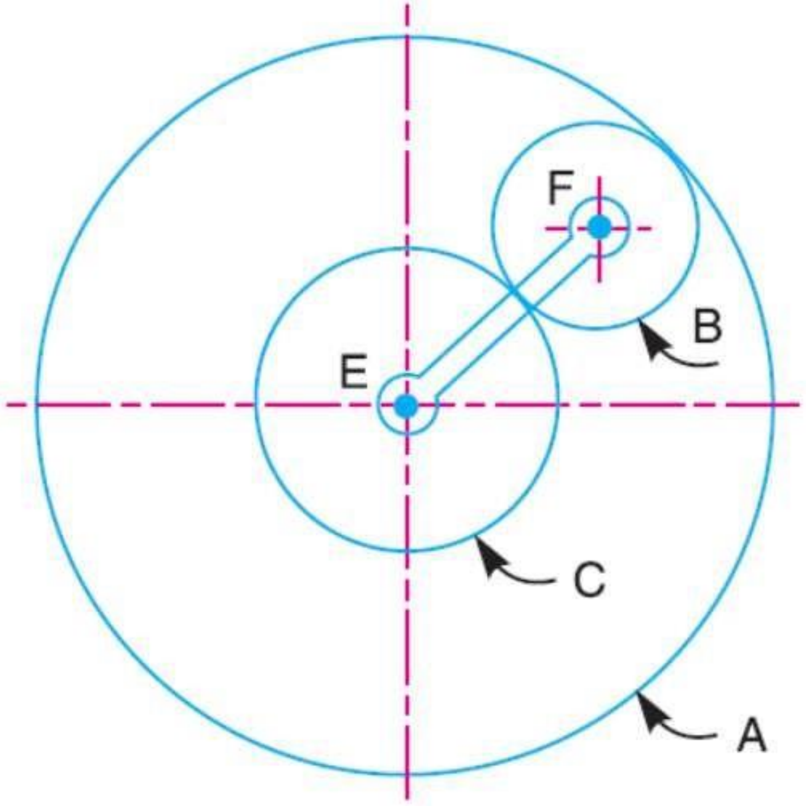


Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear D-E rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear D-E rotated through $+ x$ revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add $+ y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

<u>**400 r.p.m. (anticlockwise)**</u>

- *An epicyclic gear consists of three gears A, B and C as shown in Fig. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.*

Given : $T_A = 72$; $T_C = 32$; Speed of arm EF = 18 r.p.m.



Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear C rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B}\times\frac{T_B}{T_A}=-\frac{T_C}{T_A}$
2.	Arm fixed-gear C rotates through $+x$ revolutions	0	$+x$	$-x\times\frac{T_C}{T_B}$	$-x\times\frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_C}{T_B}$	$y-x\times\frac{T_C}{T_A}$

58.5 r.p.m 46.8 r.p.m.

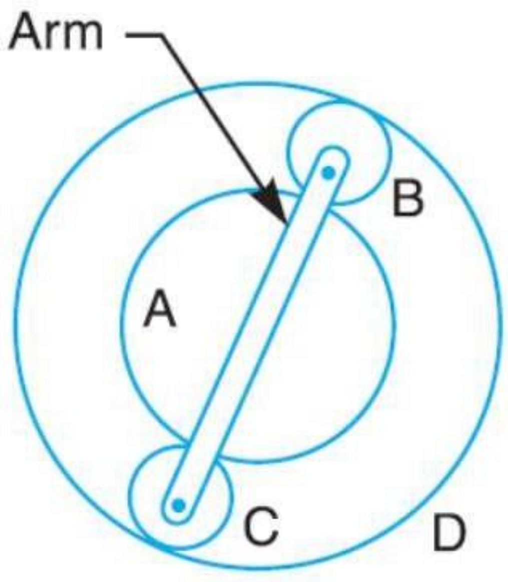
- *An epicyclic train of gears is arranged as shown in Fig.. How many revolutions does the arm, to which the pinions B and C are attached, make :*
- ***1.** when A makes one revolution clockwise and D makes half a revolution anticlockwise, and*
- ***2.** when A makes one revolution clockwise and D is stationary ?*
- *The number of teeth on the gears A and D are 40 and 90respectively.*
- Given : $TA = 40$; $TD = 90$

$d_A + d_B + d_C = d_D$ or $d_A + 2\,d_B = d_D$ ...($d_B = d_C$)

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$T_A + 2\,T_B = T_D$ or $40 + 2\,T_B = 90$

$T_B = 25$, and $T_C = 25$

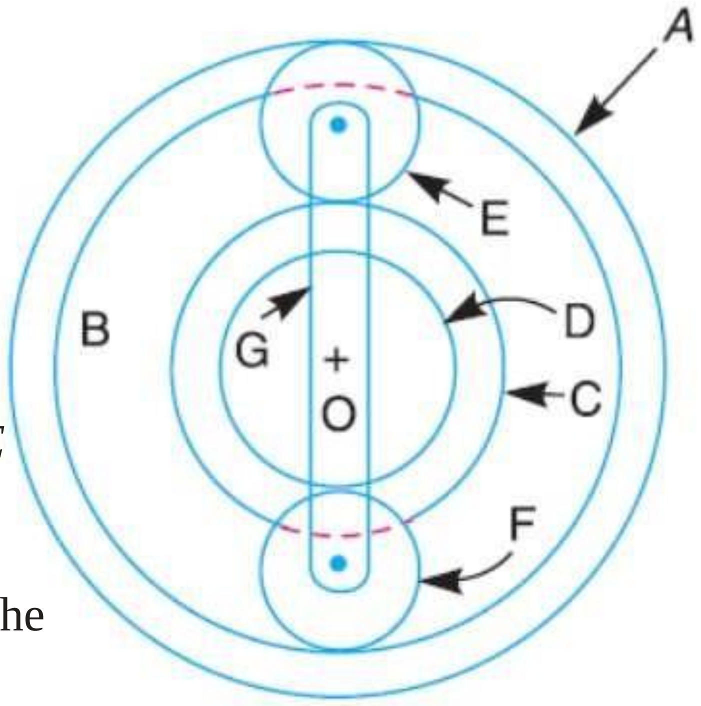


Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through $-x$ revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

1. Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise 0.04 revolution anticlockwise

2. Speed of arm when A makes 1 revolution clockwise and D is stationary Speed of arm = $-y$ = -0.308 = 0.308 revolution clockwise

- *In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth are : TC = 28; TD = 26; TE = TF = 18.* ***1.*** *Sketch the arrangement ;* ***2.*** *Find the number of teeth on A and B ;* ***3.*** *If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B ; and* ***4.*** *If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise ;find the speed of wheel B.*



Given : $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$

1. Sketch the arrangement

The arrangement is shown in Fig. 13.12.

2.Number of teeth on wheels A and B

Let T_A = Number of teeth on wheel *A*, and

T_B = Number of teeth on wheel *B*.

If d_A, d_B, d_C, d_D, d_E and d_F are the pitch circle diameters of wheels *A, B, C, D, E* and *F* respectively, then from the geometry of Fig. 13.12,

$d_A = d_C + 2\,d_E$ and $d_B = d_D + 2\,d_F$

Since the number of teeth are proportional to their pitch circle diameters, for the samemodule, therefore

$T_A = T_C + 2\,T_E = 28 + 2 \times 18 = 64$ **Ans.**

and $T_B = T_D + 2\,T_F = 26 + 2 \times 18 = 62$ **Ans.**

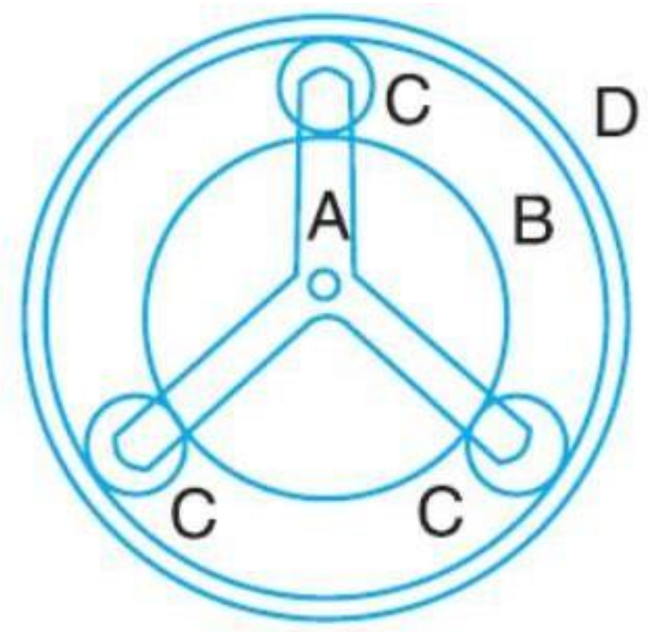
Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed- wheel A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	$+1$	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C}$ $=-\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B}$ $=+\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2.	Arm fixed-wheel A rotates through $+x$ revolutions	0	$+x$	$+x\times\frac{T_A}{T_E}$	$-x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_A}{T_E}$	$y-x\times\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

3. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A is fixed 4.2 r.p.m. clockwise

4.Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise 5.4 r.p.m. counter clockwise

- *In an epicyclic gear of the 'sun and planet' type shown in Fig. , the pitch circle diameter of the internally toothed ring is to be 224 mm and the module 4 mm. When the ring D is stationary, the spider A, which carries three planet wheels C of equal size, is to make one revolution in the same sense as the sunwheel B for every five revolutions of the driving spindle carrying the sunwheel B. Determine suitable numbers of teeth for all the wheels.*

Given : $d_D = 224$ mm ; $m = 4$ mm ; $N_A = N_B / 5$



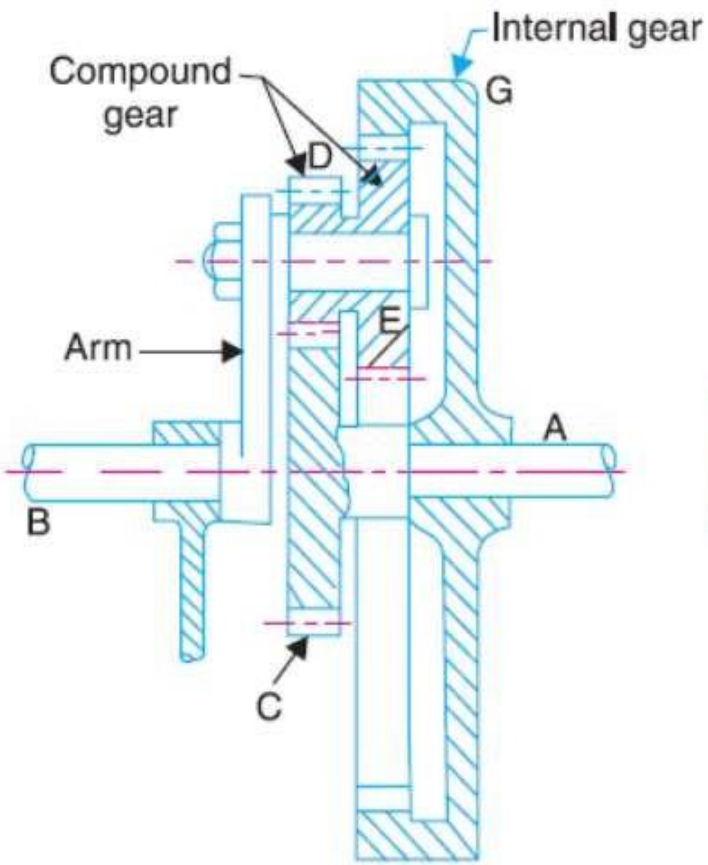
Step No.	Conditions of motion	Revolutions of elements			
		Spider A	Sun wheel B	Planet wheel C	Internal gear D
1.	Spider A fixed, sun wheel B rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_B}{T_C}$	$-\frac{T_B}{T_C}\times\frac{T_C}{T_D}=-\frac{T_B}{T_D}$
2.	Spider A fixed, sun wheel B rotates through + x revolutions	0	$+x$	$-x\times\frac{T_B}{T_C}$	$-x\times\frac{T_B}{T_D}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_B}{T_C}$	$y-x\times\frac{T_B}{T_D}$

$T_D = d_D / m = 224 / 4 = 56$ **Ans.**

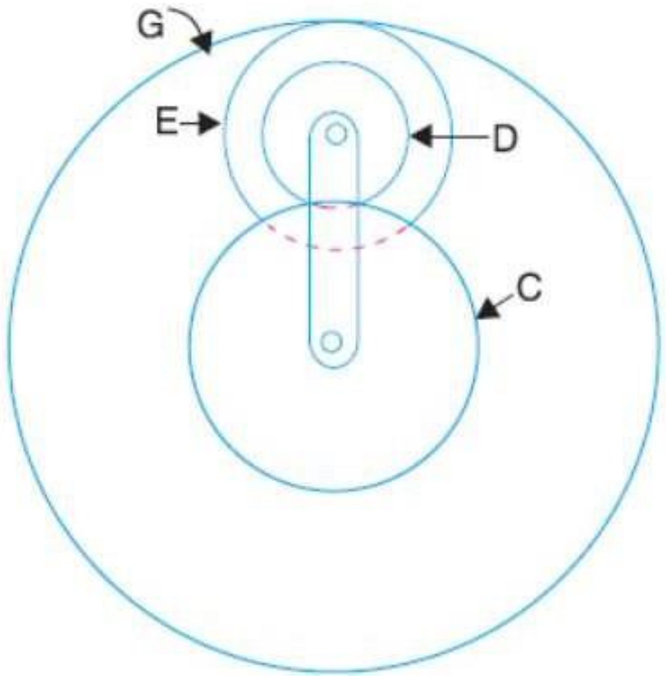
$4\ T_B = T_D / 4 = 56 / 4 = 14$ **Ans.**

$T_C = 21$ **Ans.**

- Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G, assuming that all gears have the same module. If the shaft A rotates at 110 r.p.m. , find the speed of shaft B.



Internal gear
Compound gear
G
D
E
Arm
A
B
C



G
E
D
C

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear C (or shaft A)	Compound gear D-E	Gear G
1.	Arm fixed - gear C rotates through + 1 revolution	0	+ 1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D} \times \frac{T_E}{T_G}$
2.	Arm fixed - gear C rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_D}$	$-x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_D}$	$y - x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G}$

Speed of shaft B = Speed of arm = + y = 50 r.p.m. anticlockwise

Problem 10.19. *Fig. 10.26 shows a compound epicyclic gear train in which the two gears S_1 and S_2 are integral with the input shaft, B. The arm, A_2 is integral with the output shaft, C. The planet gear P_2 revolves*

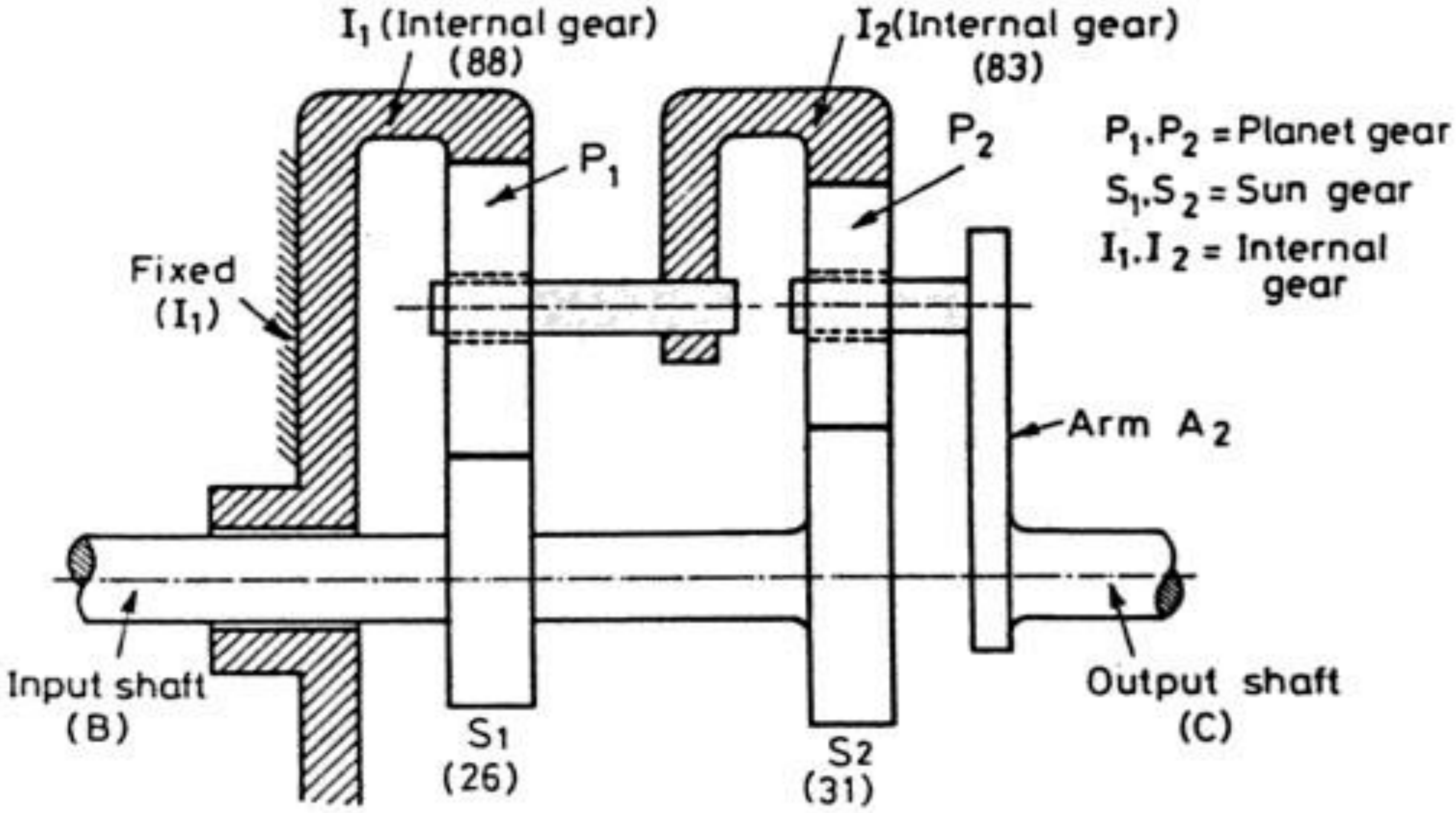


Fig. 10.26

on a pin attached to the arm A_2. The gear P_2 meshes with the sun gear S_2 and internal gear I_2. The internal gear I_2 is co-axial with the input shaft. The planet gear P_1 meshes with the fixed internal gear I_1 and sun gear S_1. The planet gear P_1 revolves on a pin fixed to internal gear I_2. The number of teeth on the gears S_1, S_2, I_1 and I_2 are 26, 31, 88 and 83 respectively. The input power on the shaft (B) is 10.47 kW at 1000 r.p.m. Find :

(i) *the speed and torque at output shaft*

(ii) *the torque required to hold the internal gear I_1 stationary. The input shaft B rotates anticlockwise.*

S. No.	Operations	Revolutions of			
		Arm A_1	Gear S_1	Gear P_1	Gear I_1
1.	Arm A_1 fixed. Gear S_1 is rotated through + 1 revolution (anticlockwise)	0	+ 1	$-\frac{T_{S_1}}{T_{P_1}}$	$-\frac{T_{S_1}}{T_{P_1}} \times \frac{T_{P_1}}{T_{I_1}} = -\frac{T_{S_1}}{T_{I_1}}$
2.	Arm A_1 fixed. Gear S_1 is rotated through $+x$ revolution (Multiply all by x)	0	$+x$	$-x \times \frac{T_{S_1}}{T_{P_1}}$	$-x \times \frac{T_{S_1}}{T_{I_1}}$
3.	Add $+y$ revolution to all	$+y$	$+y$	$+y$	$+y$
4.	Resultant motion (Add 2 and 3)	y	$x+y$	$y - x\frac{T_{S_1}}{T_{P_1}}$	$y - x\frac{T_{S_1}}{T_{I_1}}$

S. No.	*Operations*	*Revolutions of*			
		Arm A_2	*Gear S_2*	*Gear P_2*	*Gear I_2*
1.	Arm A_2 fixed. Rotate gear S_2 through + 1 revolution (anticlock-wise)	0	+1	$-\frac{T_{S_2}}{T_{P_2}}$	$-\frac{T_{S_2}}{T_{P_2}} \times \frac{T_{P_2}}{T_{I_2}} = -\frac{T_{S_2}}{T_{I_2}}$
2.	Arm A_2 fixed. Rotate gear S_2 through $+x$ revolution (Multiply by x to all)	0	$+x$	$-x\frac{T_{S_2}}{T_{P_2}}$	$-x \times \frac{T_{S_2}}{T_{I_2}}$
3.	Add $+y$ revolution to all	$+y$	$+y$	$+y$	$+y$
4.	Resultant motion (Add 2 and 3)	y	$x+y$	$y-x\frac{T_{S_2}}{T_{P_2}}$	$y-x\frac{T_{S_2}}{T_{I_2}}$

Problem 10.16. *Fig. 10.23 (figure on next page) shows an epicyclic speed reduction gear. The driving shaft is attached to an arm E. The arm carries a pin on which the compound gear B-C is free to revolve. The gear A is keyed to the driven shaft and gear D is a fixed gear. The gear B meshes with gear A whereas gear C meshes with the fixed gear D. The no. of teeth on the gears A, B, C and D are 24, 27, 30 and 21 respectively. Determine :*

(*i*) *speed of driven shaft if driving shaft is rotating at 800 r.p.m. anticlockwise and*

(*ii*) *load torque (or resisting torque) on driven shaft and holding torque on fixed gear D, if the input torque to driving shaft is 10 Nm. Assume that there is no friction losses and that the members are revolving at uniform speeds.*

Sol. Given :

$$T_A = 24\ ;\ T_B = 27\ ;\ T_C = 30 \text{ and } T_D = 21.$$

Speed of driving shaft (or speed of arm E), N_E = 800 r.p.m. anticlockwise.

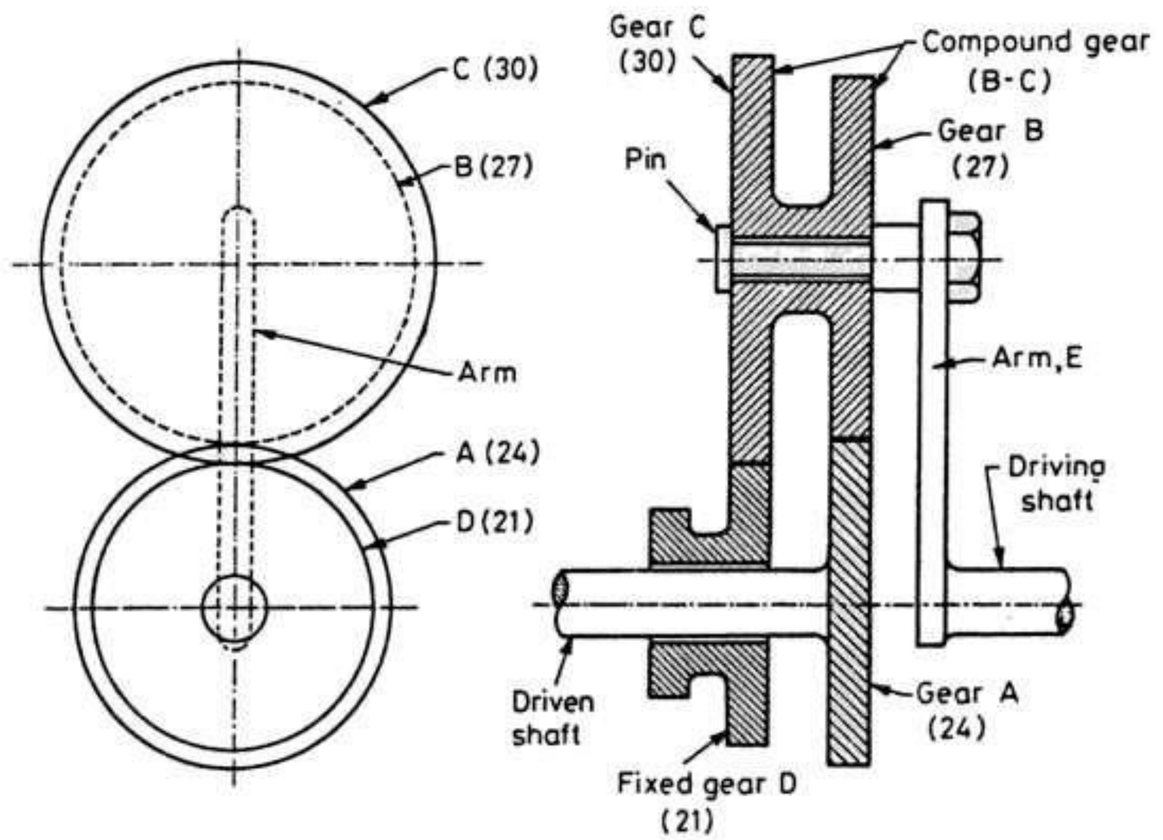
Gear D is fixed. $\quad \therefore \quad N_D = 0$

Find : (*i*) Speed of driven shaft (or speed of gear A) or N_A

(*ii*) Resisting torque (M_r) and holding torque (M_h).

(*i*) *Speed of driven shaft or speed of gear A (i.e. N_A)*

The speed of gear A can be obtained by *Tabular Method* for which the table of motion can be prepared by keeping the arm (E) fixed. It is also clear that when gear A is rotated anticlockwise, the gear B will rotate

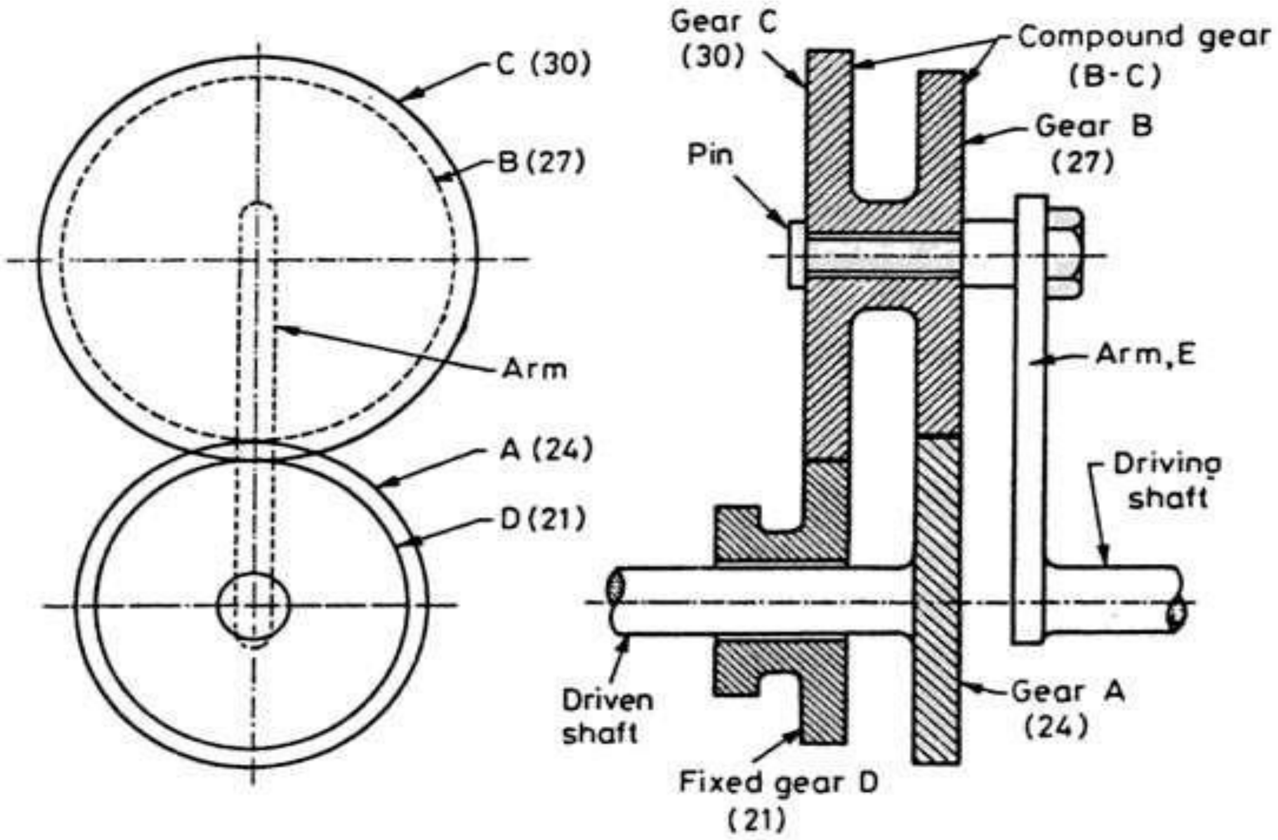

C (30)
B (27)
Arm
A (24)
D (21)
Gear C
(30)
Compound gear
(B-C)
Gear B
(27)
Pin
Arm, E
Driving
shaft
Driven
shaft
Gear A
(24)
Fixed gear D
(21)

S. No.	Operations	Revolution of			
		Arm E	Gear A	Compound gear B-C	Gear D
1.	Arm fixed. Rotate gear A through + 1 revolution	0	+ 1	$-\frac{T_A}{T_B}$	$\left(-\frac{T_A}{T_B}\right)\times\left(-\frac{T_C}{T_D}\right)$
2.	Arm fixed. Rotate gear A through + x revolution (Multiply by x to all)	0	+ x	$-x\times\frac{T_A}{T_B}$	$x\times\left(-\frac{T_A}{T_B}\right)\left(-\frac{T_C}{T_D}\right)$
3.	Add + y revolution to all	+ y	+ y	+ y	+ y
4.	Resultant Motion	+ y	x + y	$y-x\times\frac{T_A}{T_B}$	$y+x\times\frac{T_A}{T_B}\times\frac{T_C}{T_D}$

Problem 10.16. *Fig. 10.23 (figure on next page) shows an epicyclic speed reduction gear. The driving shaft is attached to an arm E. The arm carries a pin on which the compound gear B-C is free to revolve. The gear A is keyed to the driven shaft and gear D is a fixed gear. The gear B meshes with gear A whereas gear C meshes with the fixed gear D. The no. of teeth on the gears A, B, C and D are 24, 27, 30 and 21 respectively. Determine :*

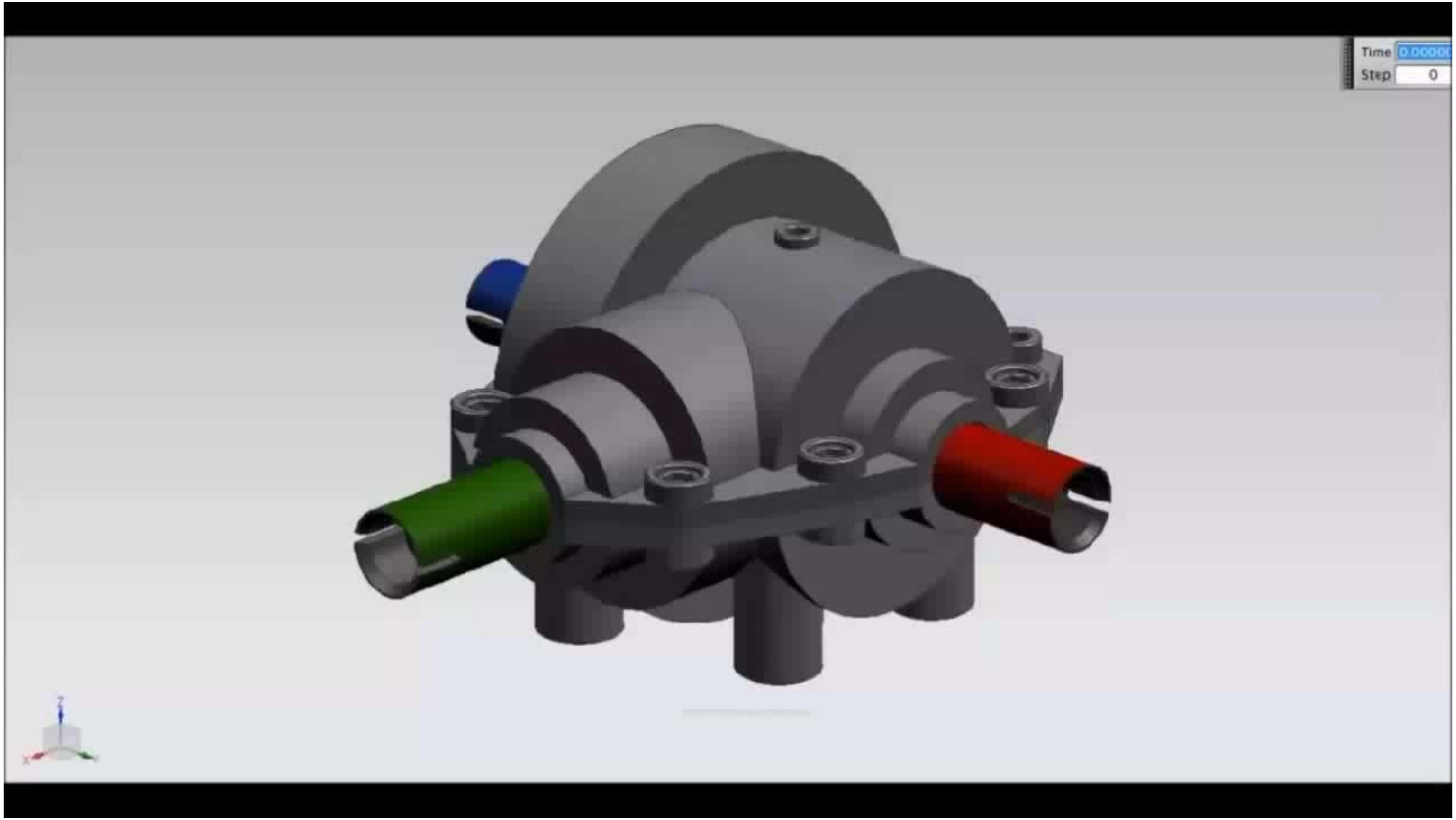
(i) speed of driven shaft if driving shaft is rotating at 800 r.p.m. anticlockwise and

(ii) load torque (or resisting torque) on driven shaft and holding torque on fixed gear D, if the input torque to driving shaft is 10 Nm. Assume that there is no friction losses and that the members are revolving at uniform speeds.

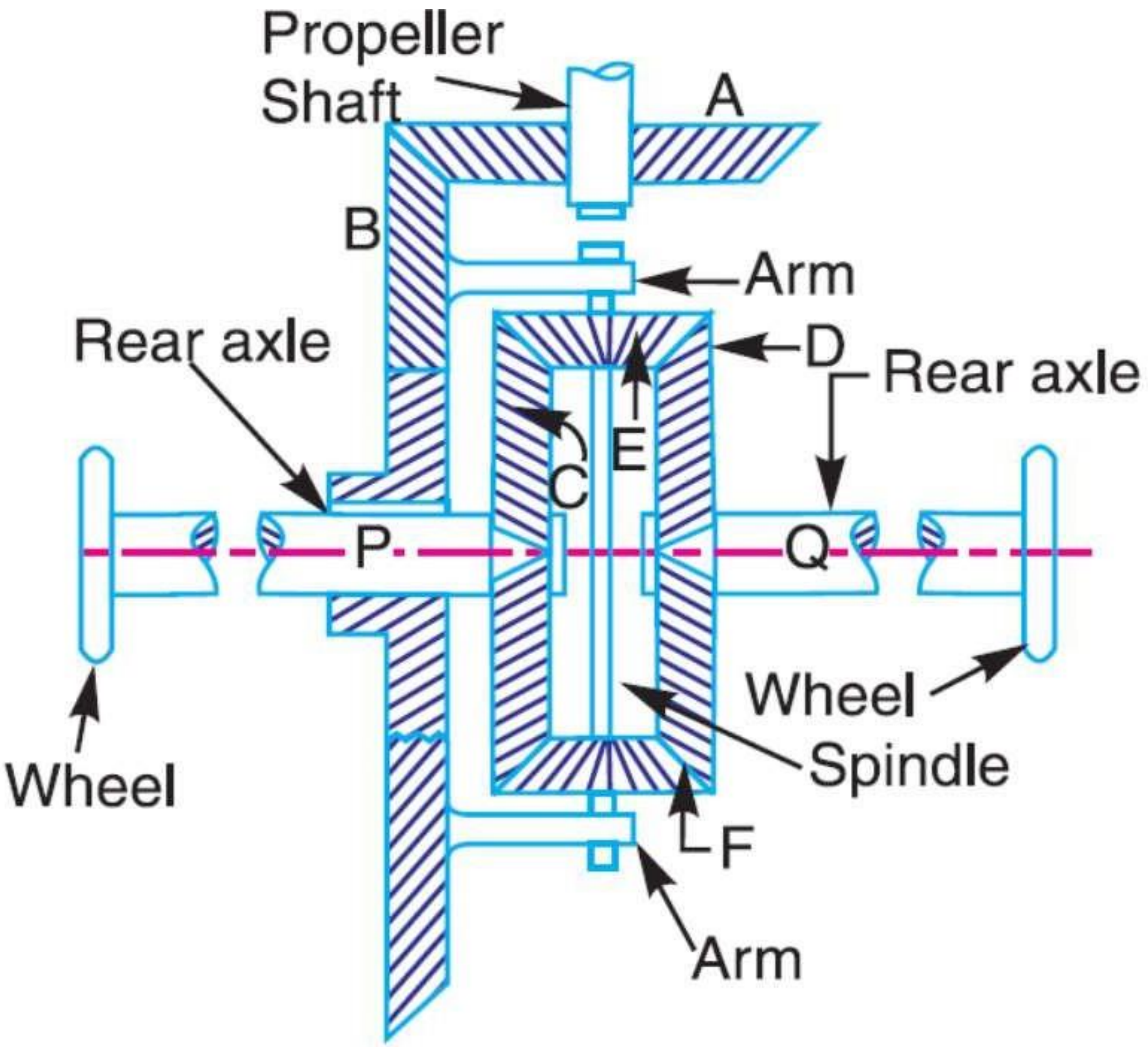

C (30)
B (27)
Arm
A (24)
D (21)
Gear C
(30)
Compound gear
(B-C)
Gear B
(27)
Pin
Arm, E
Driving
shaft
Driven
shaft
Gear A
(24)
Fixed gear D
(21)

S. No.	Operations	Revolution of			
		Arm E	Gear A	Compound gear B-C	Gear D
1.	Arm fixed. Rotate gear A through + 1 revolution	0	+ 1	$-\frac{T_A}{T_B}$	$\left(-\frac{T_A}{T_B}\right) \times \left(-\frac{T_C}{T_D}\right)$
2.	Arm fixed. Rotate gear A through + x revolution (Multiply by x to all)	0	+ x	$-x \times \frac{T_A}{T_B}$	$x \times \left(-\frac{T_A}{T_B}\right)\left(-\frac{T_C}{T_D}\right)$
3.	Add + y revolution to all	+ y	+ y	+ y	+ y
4.	Resultant Motion	+ y	x + y	$y - x \times \frac{T_A}{T_B}$	$y + x \times \frac{T_A}{T_B} \times \frac{T_C}{T_D}$

DIFFERENTIAL GEAR BOX



- ***Fig shows a differential gear used in a motor car. The pinion A on the propeller shaft has 12 teeth and gears with the crown gear B which has 60 teeth. The shafts P and Q form the rear axles to which the road wheels are attached. If the propeller shaft rotates at 1000 r.p.m. and the road wheel attached to axle Q has a speed of 210 r.p.m. while taking a turn, find the speed of road wheel attached to axle P.***



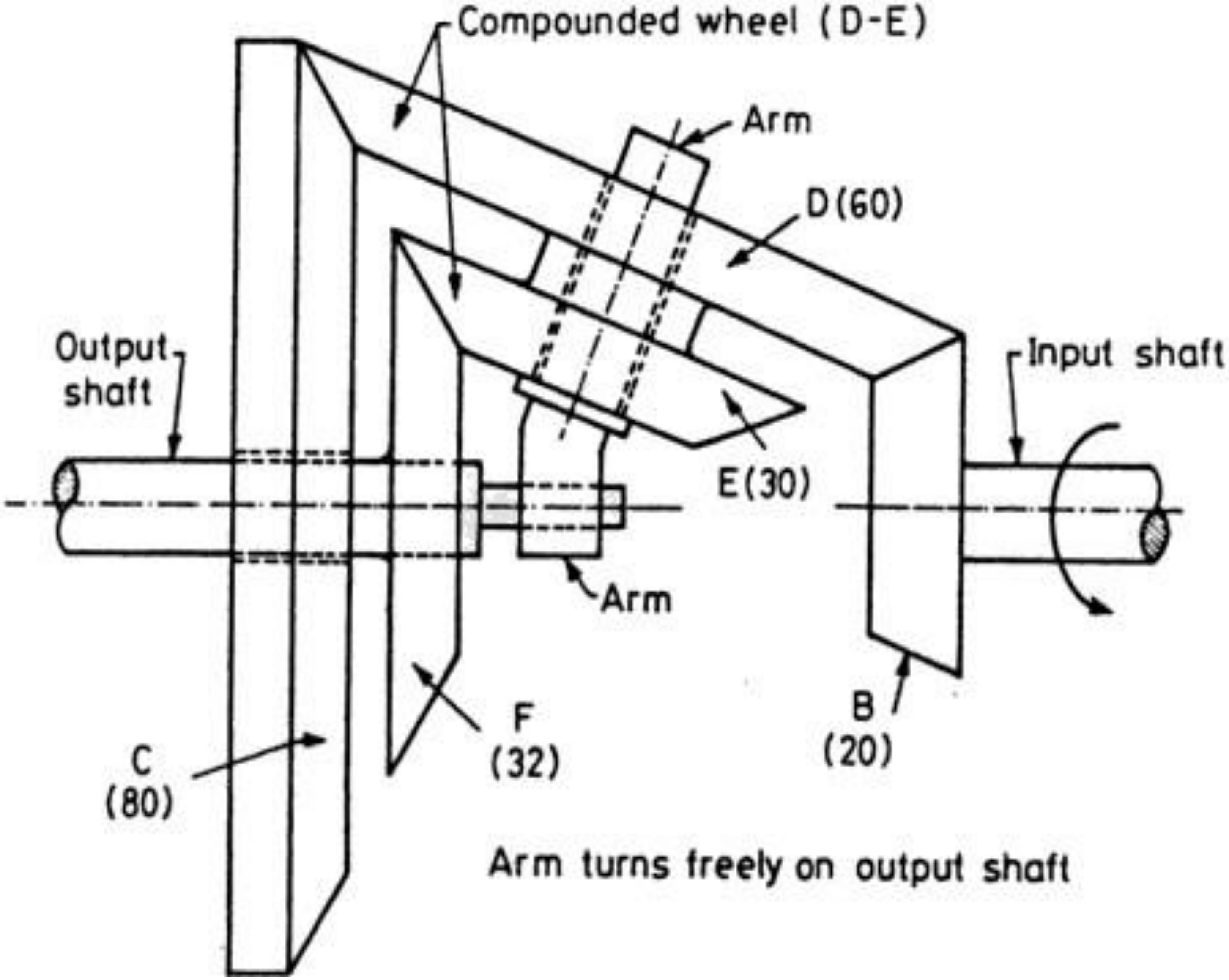
Propeller
Shaft
A
B
Arm
D
Rear axle
Rear axle
E
C
P
Q
Wheel
Wheel
Spindle
F
Arm

Step No.	Conditions of motion	Revolutions of elements			
		Gear B	Gear C	Gear E	Gear D
1.	Gear *B* fixed-Gear *C* rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	$+1$	$+\frac{T_C}{T_E}$	$-\frac{T_C}{T_E} \times \frac{T_E}{T_D} = -1$ $(\because T_C = T_D)$
2.	Gear *B* fixed-Gear *C* rotated through $+x$ revolutions	0	$+x$	$+x \times \frac{T_C}{T_E}$	$-x$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y + x \times \frac{T_C}{T_E}$	$y-x$

Speed of road wheel attached to axle P = Speed of gear $C = x + y$

$= -10 + 200 = 190$ r.p.m.

gear B is connected to the input shaft and gear F is connected to the output shaft. The arm A, carrying the compound wheels D and E, turns freely on the output shaft. If the input speed is 1000 r.p.m. anticlockwise when seen from the right, determine the speed of the output shaft when : (i) gear C is fixed, and (ii) gear C is rotated at 10 r.p.m. anticlockwise.



S. No.	Slips of Motion	Revolution of elements				
		Arm A	Gear B Input	Wheel D-E compound	Gear C	Gear F output
1.	Arm fixed, Gear B is rotated through + 1 revolution	0	+ 1	$* \frac{T_B}{T_D}$	$-\frac{T_B}{T_C}$	$-\frac{T_B}{T_D} \times \frac{T_E}{T_F}$
2.	Arm fixed, Gear B rotated through $+x$ revolution (Multiply the 1st row by x)	0	$+x$	$x\frac{T_B}{T_D}$	$-x\frac{T_B}{T_C}$	$-x\frac{T_B}{T_D} \times \frac{T_E}{T_F}$
3.	Add $+y$ revolution to all elements and get the resultant motion.	...	$x+y$	...	$-x\frac{T_B}{T_C}+y$	$-x\frac{T_B}{T_D} \times \frac{T_E}{T_F}+y$

Example 13.22. *An over drive for a vehicle consists of an epicyclic gear train, as shown in Fig. 13.29, with compound planets B-C. B has 15 teeth and meshes with an annulus A which has 60 teeth. C has 20 teeth and meshes with the sunwheel D which is fixed. The annulus is keyed to the propeller shaft Y which rotates at 740 rad /s. The spider which carries the pins upon which the planets revolve, is driven directly from main gear box by shaft X, this shaft being relatively free to rotate with respect to wheel D. Find the speed of shaft X, when all the teeth have the same module.*

When the engine develops 130 kW, what is the holding torque on the wheel D ? Assume 100 per cent efficiency throughout.

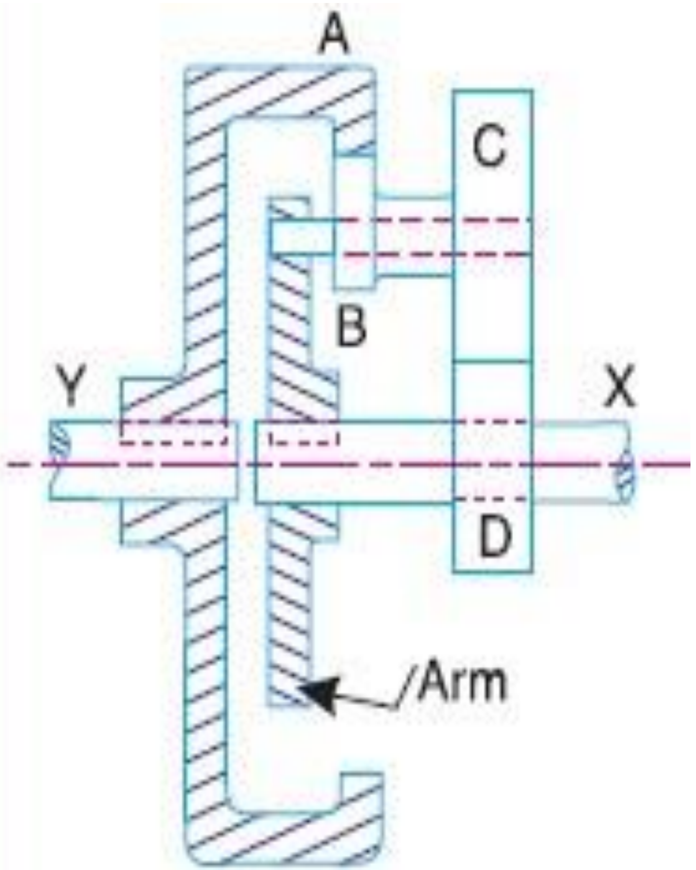


Fig. 13.29

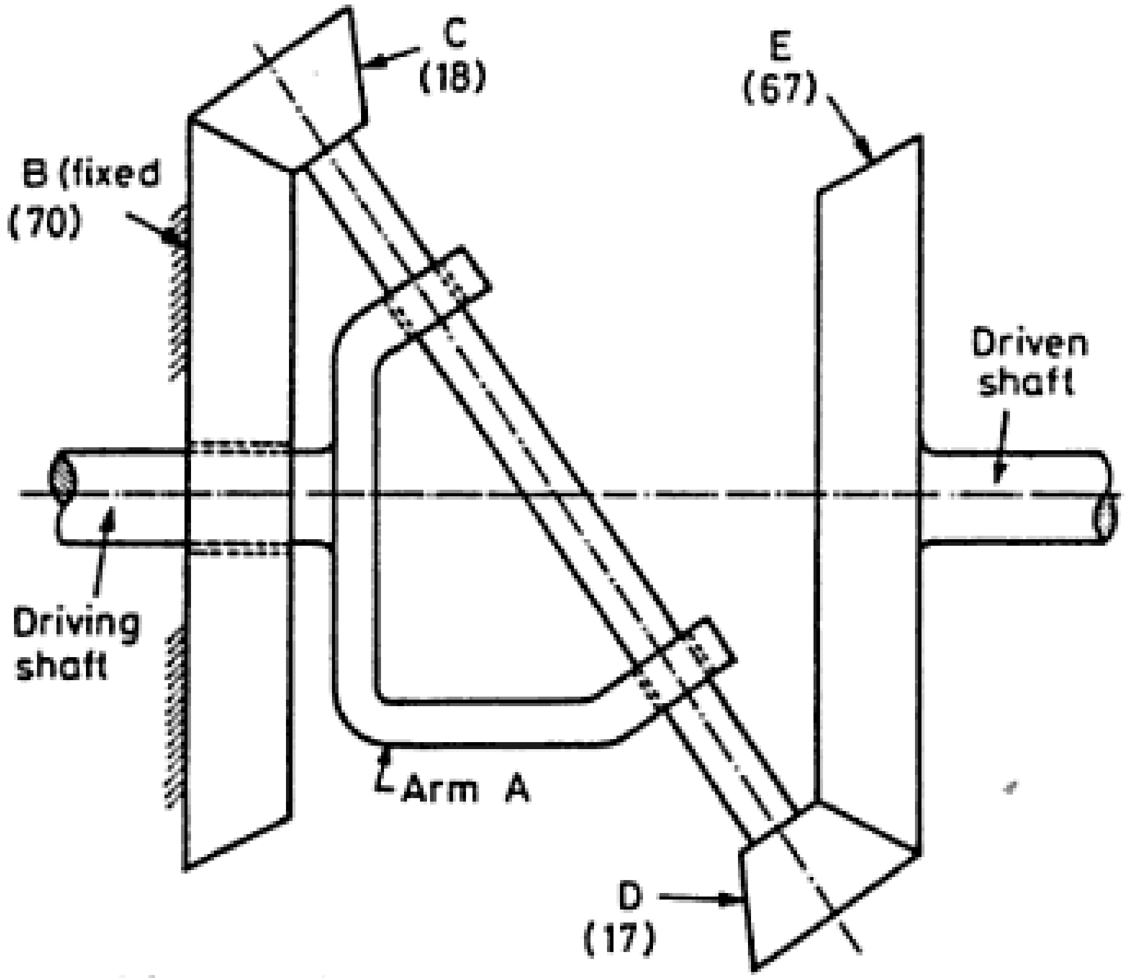
Step No.	Conditions of motion	Revolutions of elements			
		Arm (or shaft X)	Wheel D	Compound wheel C-B	Wheel A (or shaft Y)
1.	Arm fixed-wheel D rotated through + 1 revolution (anticlockwise)	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm fixed-wheel D rotated through + x revolutions	0	$+x$	$-x \times \frac{T_D}{T_C}$	$-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_D}{T_C}$	$y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$

$x = -563.8$ and $y = 563.8$

Holding torque on wheel $D = 230.6 - 175.7 = 54.9$ N-m

Problem 10.22. *Fig. 10.28 shows an epicyclic gear train in which the gear B is a fixed one and meshes with pinion C. The arm A is keyed to the driving shaft. The gear E is keyed to the driven shaft and meshes with pinion D. The pinions C and D are keyed to a shaft, which revolves in bearing on the arm A. Find the speed of the driven shaft when : (i) the driving shaft makes 400 r.p.m. ; (ii) the wheel B turns in the same sense as the driving shaft at 80 r.p.m., the driving shaft still makes 400 r.p.m.*

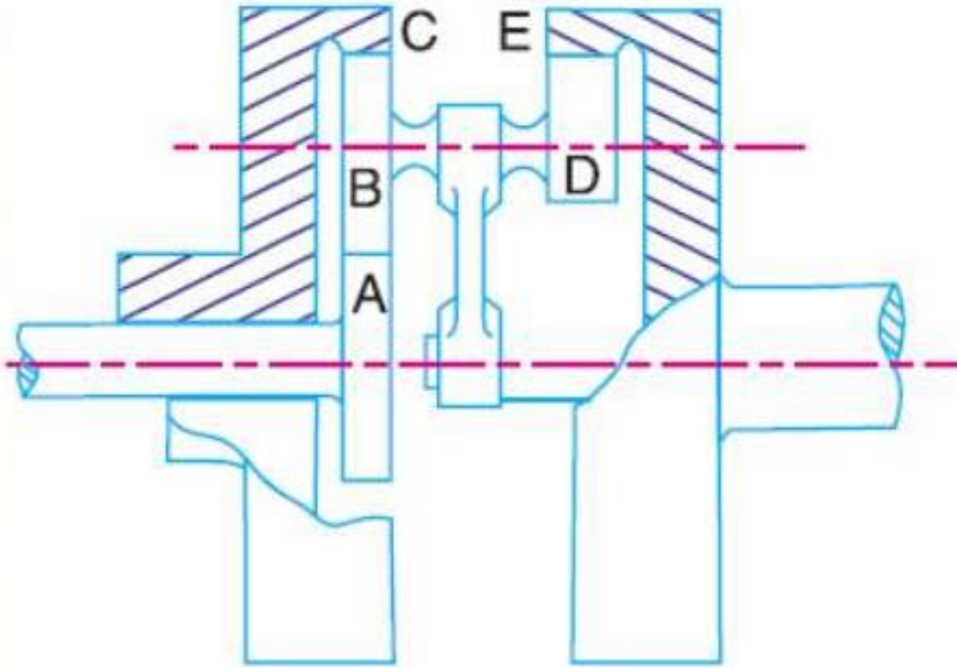
Take the number of teeth on the gears as : $T_B = 70$, $T_C = 18$, $T_D = 17$ *and* $T_E = 67$.



Pinions C & D are keyed to a shaft
which revolve in bearing on arm A

S. No.	Operations	Revolution of			
		Arm A	*Gear B*	*Pinons C-D*	*Gear E*
1.	Arm A fixed. Gear B is rotated through + 1 revolution	0	+ 1	$\frac{T_B}{T_C}$	$\frac{T_D}{T_E} \times \frac{T_B}{T_C}$
2.	Arm A fixed. Gear B is rotated through $+x$ revolution (Multiply by x)	0	$+x$	$x \times \frac{T_B}{T_C}$	$x \times \frac{T_D}{T_E} \times \frac{T_B}{T_C}$
3.	Add $+y$ revolution to all elements	$+y$	$+y$	...	$+y$
4.	Resultant motion	$+y$	$x+y$	...	$y + x\frac{T_C}{T_E} \times \frac{T_B}{T_C}$

Example 13.21. *In the epicyclic gear train, as shown in Fig. 13.28, the driving gear A rotating in clockwise direction has 14 teeth and the fixed annular gear C has 100 teeth. The ratio of teeth in gears E and D is 98 : 41. If 1.85 kW is supplied to the gear A rotating at 1200 r.p.m., find :* **1.** *the speed and direction of rotation of gear E, and* **2.** *the fixing torque required at C, assuming 100 per cent efficiency throughout and that all teeth have the same pitch.*



Step No.	Conditions of motion	Revolutions of elements				
		Arm	Gear A	Compound gear B-D	Gear C	Gear E
1.	Arm fixed-Gear A rotated through – 1 revolution (*i.e.* 1 revolution clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_C}$ $=+\frac{T_A}{T_C}$	$+\frac{T_A}{T_B}\times\frac{T_D}{T_E}$
2.	Arm fixed-Gear A rotated through – x revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_B}\times\frac{T_D}{T_E}$
3.	Add – y revolutions to all elements	$-y$	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-y-x$	$-y+x\times\frac{T_A}{T_B}$	$-y+x\times\frac{T_A}{T_C}$	$-y+x\times\frac{T_A}{T_B}\times\frac{T_D}{T_E}$

Fixing torque required at *C* = 4416 – 14.7 = 4401.3 N-m

Example 13.22. *An over drive for a vehicle consists of an epicyclic gear train, as shown in Fig. 13.29, with compound planets B-C. B has 15 teeth and meshes with an annulus A which has 60 teeth. C has 20 teeth and meshes with the sunwheel D which is fixed. The annulus is keyed to the propeller shaft Y which rotates at 740 rad /s. The spider which carries the pins upon which the planets revolve, is driven directly from main gear box by shaft X, this shaft being relatively free to rotate with respect to wheel D. Find the speed of shaft X, when all the teeth have the same module.*

When the engine develops 130 kW, what is the holding torque on the wheel D ? Assume 100 per cent efficiency throughout.

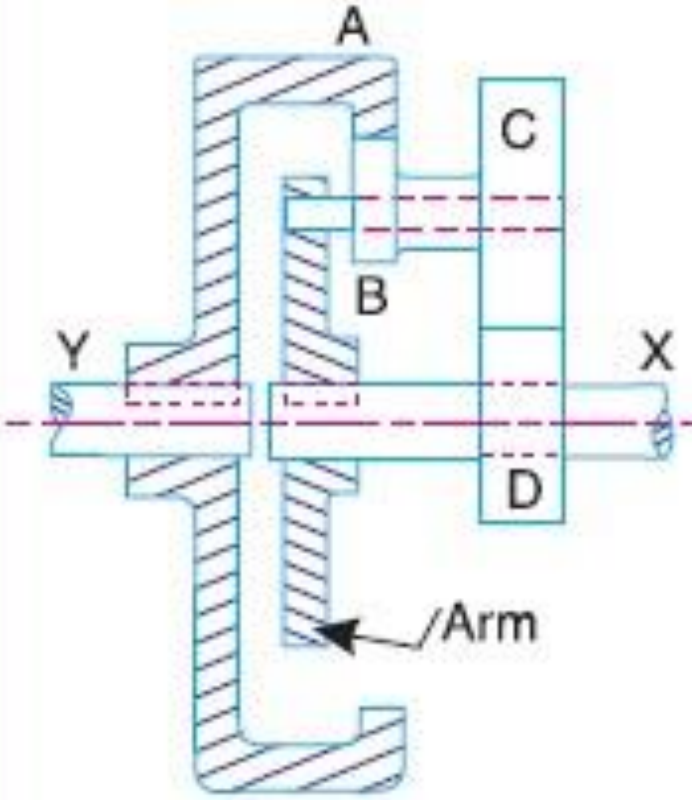


Fig. 13.29

- THANK YOU